Plasma flows near a reconnecting current layer: strong magnetic field approximation

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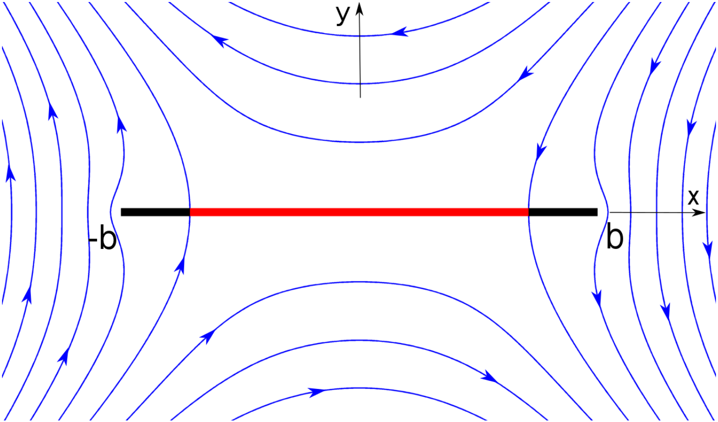
- ***2D*** velocity field $\mathbf{v}(x, y, t)$.
- ***Strong*** magnetic field $\mathbf{B}(x, y, t)$.
- The density $\rho(x, y, t)$.



The ideal MHD system of partial differential equations

$$\frac{d\mathbf{v}}{dt} = -\frac{\nabla p}{\rho} - \frac{1}{4\pi\rho}[\mathbf{B} \times rot\mathbf{B}]$$

$$\frac{\partial \mathbf{B}}{\partial t} = rot[\mathbf{v} \times \mathbf{B}]$$

$$\frac{\partial \rho}{\partial t} + div\rho\mathbf{v} = 0$$

The dimensionless equations and the approximation used

$$\frac{\varepsilon^2}{\delta}\frac{\partial \mathbf{v}}{\partial t} + \varepsilon^2(\mathbf{v}\nabla)\mathbf{v} = -\gamma^2\frac{\nabla p}{\rho} - \frac{1}{\rho}[\mathbf{B} \times rot\mathbf{B}],$$

$$\frac{\partial \mathbf{B}}{\partial t} = \delta rot[\mathbf{v} \times \mathbf{B}], \quad \frac{\partial \rho}{\partial t} + \delta div \rho\mathbf{v} = 0,$$

where

$$\delta = \frac{VT}{L} \simeq 1, \quad \varepsilon^2 = \frac{v^2}{V_A^2}, \quad \gamma^2 = \frac{p_0^2}{\rho_0 V_A^2}.$$

Strong field and cold plasma $\quad \gamma^2 << \varepsilon^2 << 1.$

The equations in the zeroth-order in ε^2

$$
\begin{cases}
rot\mathbf{B} = \mathbf{0}, & (1) \\
\mathbf{B}\dfrac{d\mathbf{v}}{dt} = 0, & (2) \\
\dfrac{\partial \mathbf{B}}{\partial t} = rot[\mathbf{v} \times \mathbf{B}], & (3) \\
\dfrac{\partial \rho}{\partial t} + div\,\rho\mathbf{v} = 0. & (4)
\end{cases}
$$

2D approximation ([1])

We consider 2D magnetic fields and plasma motions:

$$\mathbf{v} = \{v_x(x, y, t), v_y(x, y, t), 0\},$$

$$\mathbf{B} = \{B_x(x, y, t), B_y(x, y, t), 0\},$$

$$\mathbf{j} = \{0, 0, j(x, y, t)\},$$

$$\mathbf{A} = \{0, 0, A(x, y, t)\}.$$

[1] Somov B.V., Plasma Astrophysics, Part I, Fundamentals and Practice, Second Edition, Springer SBM, 2012, New York

2D approximation

$$F = A(x, y, t) + iA^+(x, y, t),$$

$$A^+(x, y, t) = \int (-\frac{\partial A}{\partial y} dx + \frac{\partial A}{\partial x} dy) + A^+(t),$$

$$z = x + iy,$$

$$\mathbf{B} = B_x + iB_y = -i\overline{\frac{dF}{dz}}.$$

Set of the equations in terms of $A(x, y, t)$

$$
\left\{
\begin{array}{l}
(1) \\
(2) \\
(3) \\
(4)
\end{array}
\right.
\Longrightarrow
\left\{
\begin{array}{lr}
\triangle A = 0, & (5) \\
[\frac{d\mathbf{v}}{dt} \times \nabla A] = \mathbf{0}, & (6) \\
\frac{dA}{dt} = 0, & (7) \\
\frac{\partial \rho}{\partial t} + div\, \rho \mathbf{v} = 0. & (8)
\end{array}
\right.
$$

Plasma kinematics

According to the previous system the plasma kinematics is described by the following ordinary differential equations

$$\begin{cases} \ddot{x}\dfrac{\partial A}{\partial y} - \ddot{y}\dfrac{\partial A}{\partial x} = 0, & \textbf{(6)} \\[2ex] \dot{x}\dfrac{\partial A}{\partial x} + \dot{y}\dfrac{\partial A}{\partial y} + \dfrac{\partial A}{\partial t} = 0. & \textbf{(7)} \end{cases}$$

The Syrovatskii current layer

The complex potential of the Syrovatskii CL is given by the following formula ([2])

$$F(z,t) = \frac{\alpha}{2} z \sqrt{z^2 - b^2} + \frac{\Gamma}{2\pi} Ln \frac{z + \sqrt{z^2 - b^2}}{b} + A(t)$$

where

$$\Gamma = -\pi \alpha b^2 \nu \qquad \nu \in [0,1]$$

is the coefficient proportional to the full current in the layer.

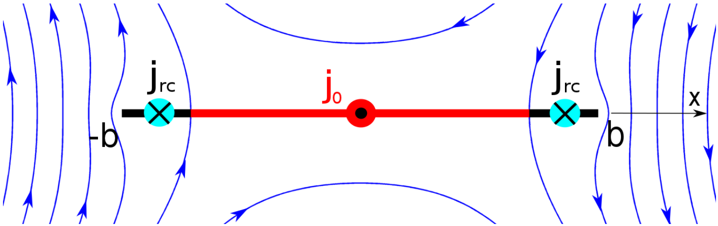
[2]Syrovatskii, Soviet Physics JETP, 1971, Volume 33, № 5

Stationary current layer

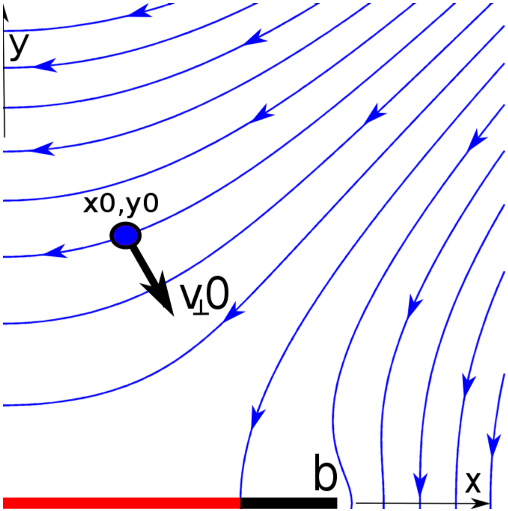
The CL with permanent width of $2b$

$$A(x,y,t) = A(x,y) + A(t), \quad A(t) = A(0) - t,$$

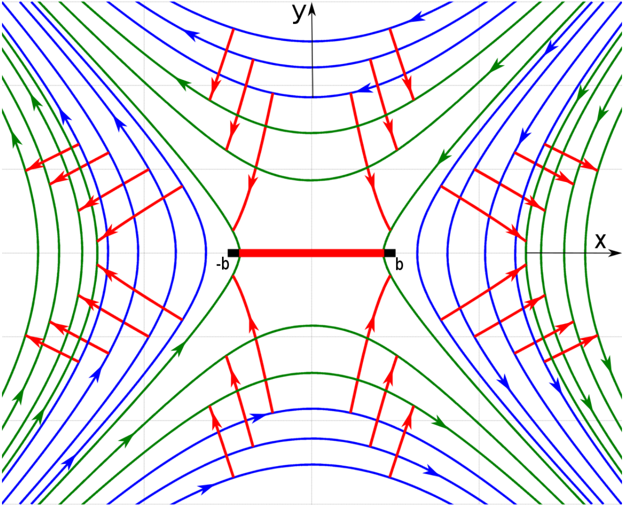
$$A(0) = \frac{b^2}{4} - \nu \frac{b^2}{2} ln \frac{b}{2l}.$$



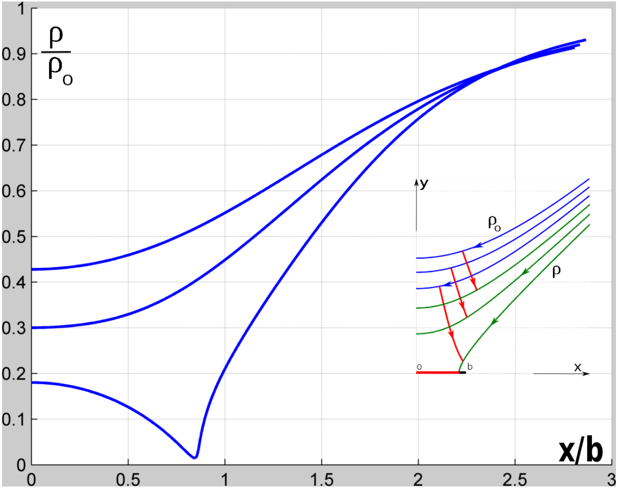
Initial conditions



Numerical results: flow pattern

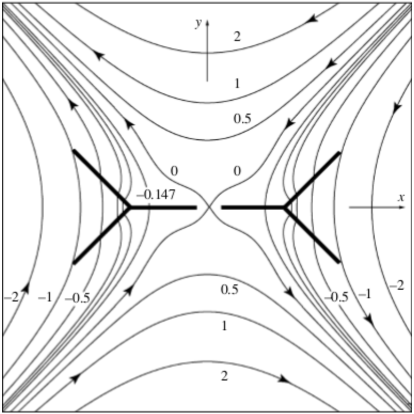


Density distribution along the field lines



The RCL with attached MHD shocks

The method allows to calculate the velocity field in a more general case:



[3] Bezrodnykh, Vlasov, Somov, Astronomy Letters, **37** (2), 113-130, 2011.

Conclusions

• The method allows to solve a system of ordinary differential equations instead of partial differential equations system.

• The method used is applicable for the investigation of the generalized model of a reconnecting current layer with attached MHD discontinuities.