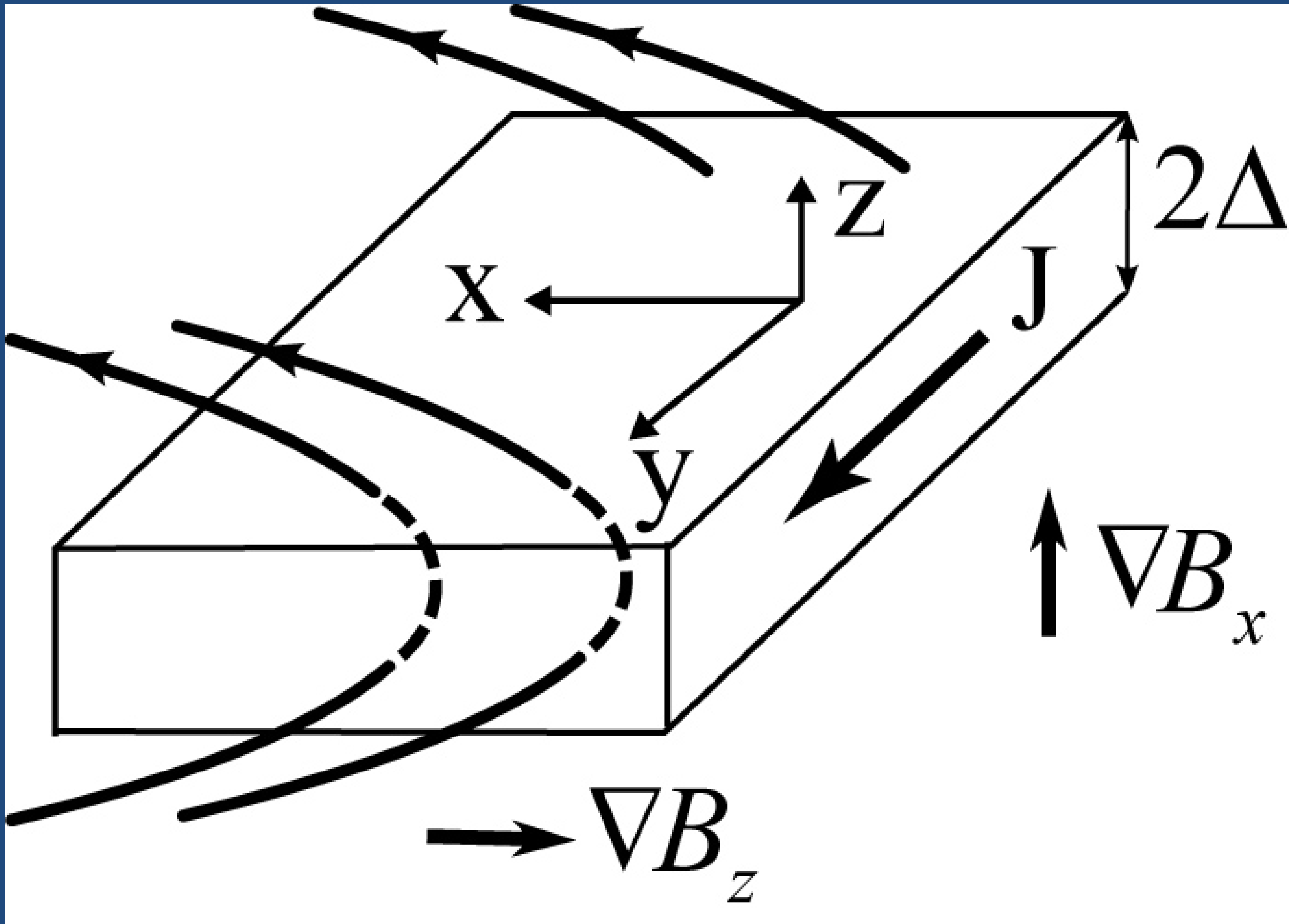
МГД МОДЕЛИРОВАНИЕ МАГНИТНОЙ ДАБЛ-ГРАДИЕНТ (ФЛЭППИНГ) НЕУСТОЙЧИВОСТИ В МАГНИТНЫХ КОНФИГУРАЦИЯХ ТИПА ХВОСТА ЗЕМНОЙ МАГНИТОСФЕРЫ С НУЛЕВОЙ/НЕНУЛЕВОЙ ТРАНСВЕРСАЛЬНОЙ МАГНИТНОЙ КОМПОНЕНТОЙ

Коровинский[1] Д., Иванов[1] И., Семенов[1] В., Еркаев [2,3] Н., Артемьев[4] А., Дивин[5] А., Иванова[1] В., Кубышкина[1] Д.

1. Санкт-Петербургский Государственный Университет, Санкт-Петербург, Россия;
2. Институт компьютерного моделирования СОРАН, Красноярск, Россия;
3. Сибирский Федеральный Университет , Красноярск, Россия;
4. Институт Космический Исследований РАН, Москва, Россия;
5. Шведский институт космической физики, Упсала, Швеция;

ИКИ, 2014

Introduction: configuration

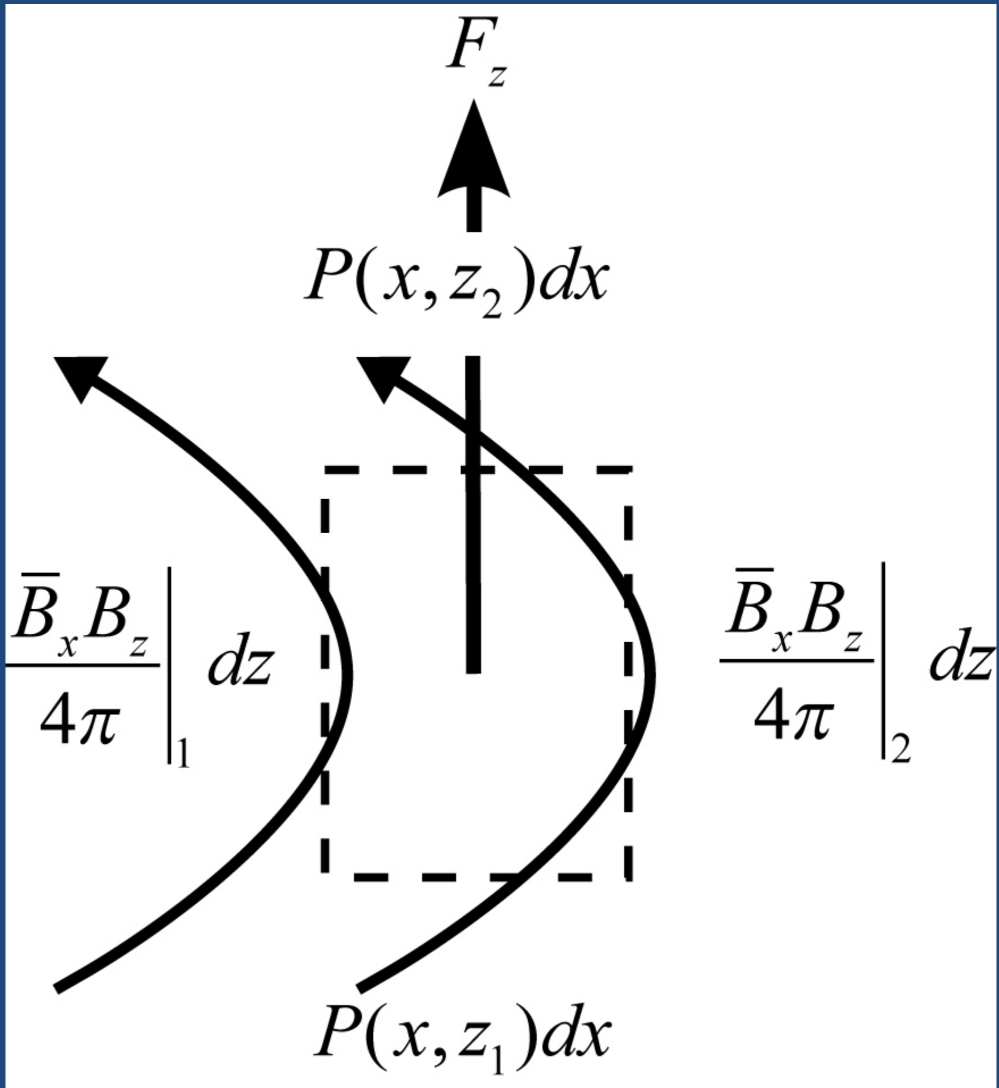


Sketch of the magnetotail current sheet and magnetic field lines.
Feature: two magnetic gradients

Wavelength band:

$$R_c < \lambda < L$$

Introduction: equilibrium



A plasma element in the center of the current sheet

In equilibrium state

$$\frac{\partial P}{\partial z} = \frac{1}{4\pi} B_x \frac{\partial B_z}{\partial x}$$

Displacement along the Z axis yields the restoring force

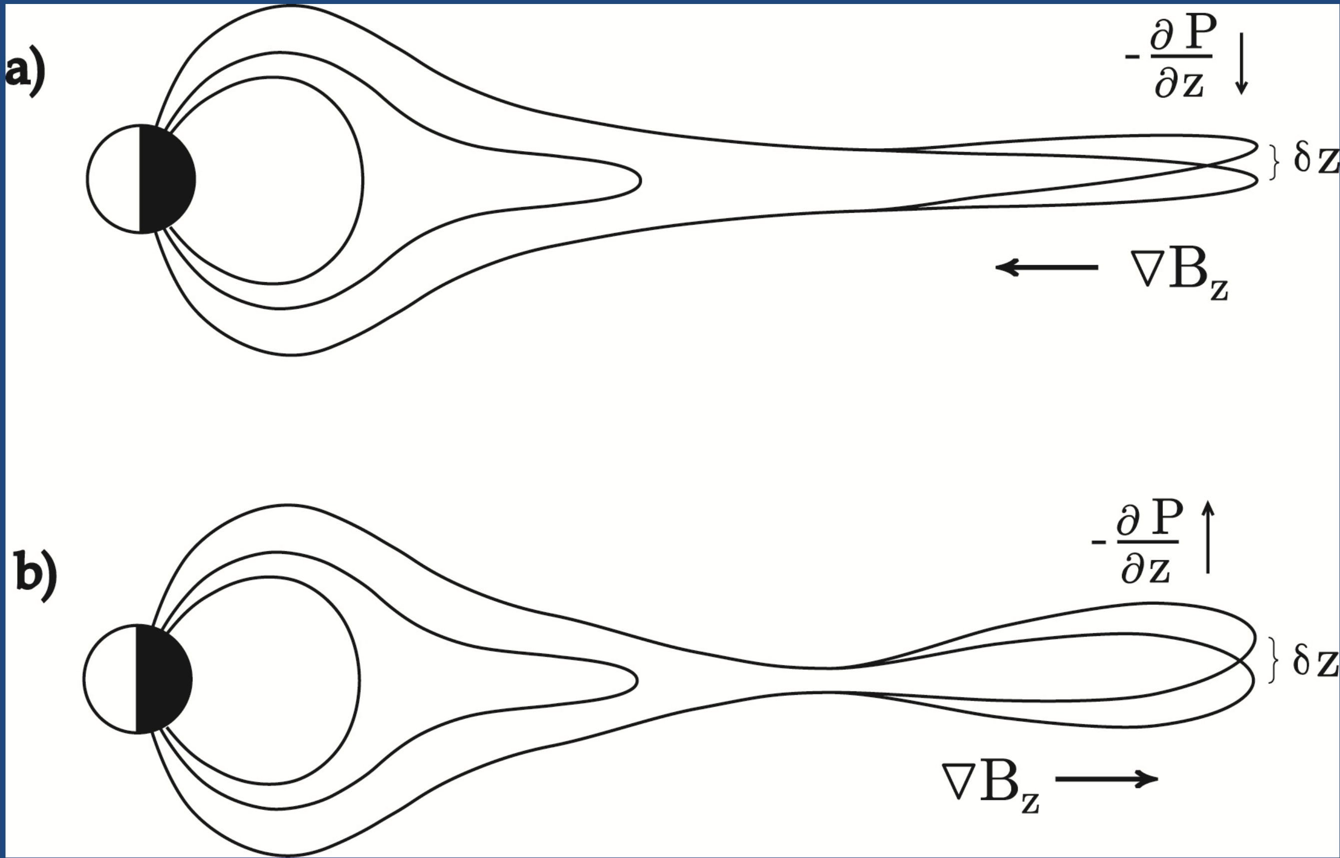
$$F_z = -\frac{1}{4\pi}\delta z \left(\frac{\partial B_x}{\partial z} \frac{\partial B_z}{\partial x} \right)_{z=0}$$

Equation of motion of the plasma element

$$\frac{\partial^2 \delta z}{\partial t^2} = -\omega_f^2 \delta z,$$

$$\omega_f^2 = \left\langle \frac{1}{4\pi\rho} \frac{\partial B_x}{\partial z} \frac{\partial B_z}{\partial x} \right\rangle_{z=0}$$

Introduction: (in)stability



$$\omega_f^2 > 0$$

Oscillations

$$\omega_f^2 < 0$$

Wave growth

a) Stable situation, minimum of the total pressure in the center of the sheet, oscillations

b) Unstable situation, maximum of the total pressure, exponential growth of initial perturbation

Analytical solution of Erkaev et al. [2007]

System of MHD equations

$$\rho \frac{d\mathbf{V}}{dt} + \nabla P = \frac{1}{4\pi\rho}(\mathbf{B}\cdot\nabla)\mathbf{B},$$

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{B} = (\mathbf{B}\cdot\nabla)\mathbf{V},$$

$$\nabla\cdot\mathbf{V} = 0, \qquad \nabla\cdot\mathbf{B} = 0.$$

Normalization

$$B^*, \quad \rho^*, \quad \Delta\&\mathrm{L}, \quad P^* = \frac{B^{*2}}{4\pi},$$

$$V_A = \frac{B^*}{\sqrt{4\pi\rho^*}}, \qquad t^* = V_A/\Delta$$

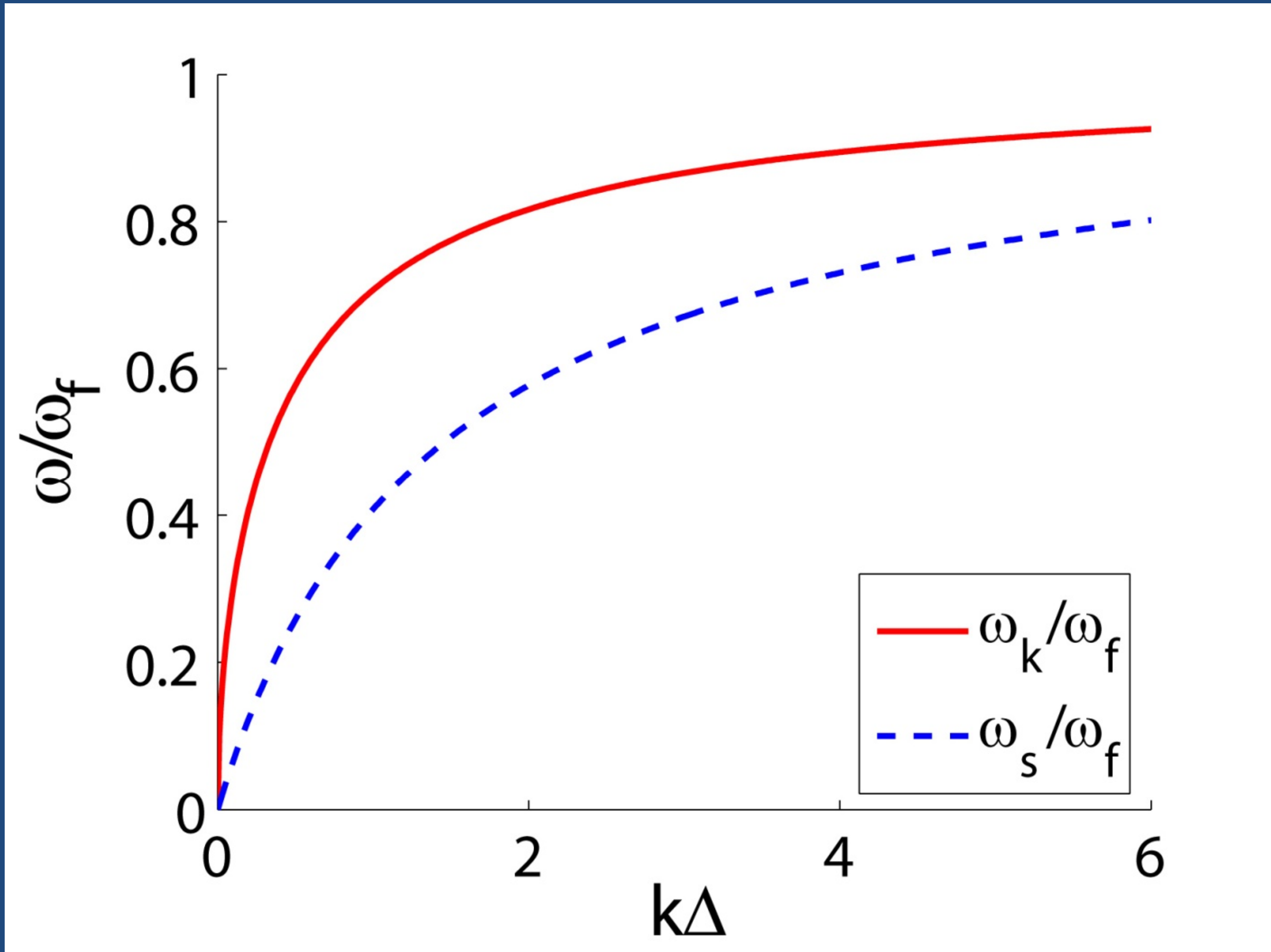
Simplifying assumptions

- incompressibility
- $\mathrm{B} = [\ \mathrm{B_x}(z/\Delta),\ 0,\ \mathrm{B_z}(x/L)\ ]$

  $B_x = \tanh(z), \qquad B_z = a + bx$
- $\varepsilon = \mathrm{B_z}(0)/\mathrm{B_{x\,max}} << 1$

  valid for Kan(1973)-like equilibrium
- $\nu = \Delta/\mathrm{L} << 1$
- $\varepsilon/\nu = (\mathrm{B_z}/\Delta)\ /\ (\mathrm{B_x}/\mathrm{L}) << 1$
- independence of perturbations on X

Analytical solution of Erkaev et al. [2007]



Dispersion curve of the kink mode
($\gamma = \mathrm{Im}[\omega]$ – growth rate)

$$\omega_k = \omega_f \sqrt{\frac{k\Delta}{k\Delta + 1}}$$

Generalization (Kub.): non-zero B_y, ρ=const

MHD System

$$\rho\left(\frac{\partial\vec{v}}{\partial t}+(\vec{v}\cdot\nabla)\vec{v}\right)=-\nabla P+\frac{1}{4\pi}\nabla\times\vec{B}\times\vec{B},$$

$$\frac{\partial\vec{B}}{\partial t}=\nabla\times\vec{v}\times\vec{B},$$

$$\left(\nabla\cdot\vec{B}\right)=0,\qquad\left(\nabla\cdot\vec{v}\right)=0.$$

Assume perturbations to be ~*exp[i(ky-ωt)]*

Normalization:

$$B\rightarrow\frac{B}{B_0},\quad L\rightarrow\frac{L}{\Delta},\quad T\rightarrow T\omega_f,$$

$$v\rightarrow\frac{v}{\Delta\cdot\omega_f},\quad \omega_f^2=\frac{|\partial_z B_{0x}\cdot\partial_x B_{0z}|}{4\pi\rho},$$

$$(\mathrm{x,y,z})\rightarrow(\mathrm{x,y,z})/\Delta,\quad \rho=const.$$

Background magnetic configuration:

$$B_0=[B_{0x},B_{0y},B_{0z}],$$

$$B_{0x}=\begin{cases}Cz, & |z|\le\Delta,\\ C\Delta, & |z|>\Delta.\end{cases}\qquad B_{0z}=a+bx;\qquad B_{0y}=const,\qquad C=const.$$

Generalization (Kub.): non-zero B_y, ρ=const

Equation for the displacement:

$$\frac{\partial^2 \vec{\xi}_z}{\partial z^2} + \underbrace{k^2 \left(\frac{\omega_f^2 + \omega^2 + k^2 V_{Ay}^2}{\omega^2 + k^2 V_{Ay}^2} \right)}_{\lambda^2} \vec{\xi}_z = 0, \qquad \vec{v}_z = \frac{\partial \vec{\xi}_z}{\partial t}$$

Quantities $\xi_z,\ \partial \xi_z / \partial t$ are continuous across the sheet, fading toward the flanks. For the sheet of the finite width (2Δ) the tractability condition yields

$$\left.\begin{aligned} \frac{k}{\lambda} &= tg(\lambda), && \text{kink} \\ \frac{k}{\lambda} &= -ctg(\lambda), && \text{sausage} \end{aligned}\right\} \Rightarrow \lambda(k) \Rightarrow \qquad \boxed{\gamma = \mathrm{Im}[\omega] = k\sqrt{\frac{\omega_f^2}{\lambda^2 + k^2} - V_{Ay}^2}}$$

Generalization (Art.): non-zero B_y, $\rho \neq$ const

MHD System

$$\begin{cases} \rho\left(\dfrac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{V}\right) + \nabla P = \dfrac{1}{4\pi}\mathbf{B}\cdot\nabla\mathbf{B} \\ \dfrac{\partial \mathbf{B}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{B} = \mathbf{B}\cdot\nabla\mathbf{V} \\ \dfrac{\partial \rho}{\partial t} + \mathbf{V}\cdot\nabla\rho = 0, \quad \nabla\mathbf{B} = 0 \end{cases}$$

Initial Equilibrium

[*Kan, 1973, JGR; Birn et al., 1975, SSR; Lembege and Pellat, 1982, Phys. Fluids*]

$$\mathbf{B} = B_0(x)\tanh\left(z / L(x)\right)\mathbf{e}_x + B_z(x,z)\mathbf{e}_z$$

$$+B_y\mathbf{e}_y$$

Local approximation

$$\mathbf{B} \approx B_0 \tanh\left(z / L_{eff}\right)\mathbf{e}_x + B_z(x)\mathbf{e}_z + B_y\mathbf{e}_y, \quad L_{eff} \approx \left\langle L(x)\right\rangle_x, \quad B_z(x) \approx B_z(x, z=0)$$

Perturbations and normalized variables

$$\left\{\mathbf{b}_1, \mathbf{V}_1, \rho_1\right\} \sim \exp(iky - \omega t)$$

$$\mathbf{b} = \mathbf{B} / B_0, \quad \mathbf{r} \to \mathbf{r} / L_{eff}, \quad \rho \to \rho / \rho(z=0), \quad K = kL_{eff}$$

$$\Omega = \omega L_{eff} / v_A, \quad \mathbf{u} = \mathbf{V}_1 / v_A, \quad v_A = B_0 / \sqrt{4\pi\rho(z=0)}$$

$$b_n = B_z / B_0 \approx const$$

$$\partial B_z / \partial x = \nu b_n' \approx const$$

$$b_m = B_y / B_0 \approx const$$

$$\nu b_n' \gg b_n^2, \quad \nu b_n' b_n \sim b_n^2,$$

Generalization (Art): disp. relation with $B_y = 0$

Equation for the u_z velocity component (compressible plasma)

$$\frac{1}{\rho}\frac{d}{dz}\left(\rho\frac{du_z}{dz}\right)+K^2u_z\left(U(z)-1\right)=0,\quad U(z)=\frac{\nu b_n'}{\Omega^2}\frac{1}{\rho(z)}\frac{db_{0x}(z)}{dz}$$

Erkaev et al., 2009, AnGeo

$$\begin{cases} du_z/dz=0, & z=0 \\ u_z\sim\exp(-Kz), & z\to\infty \end{cases}$$

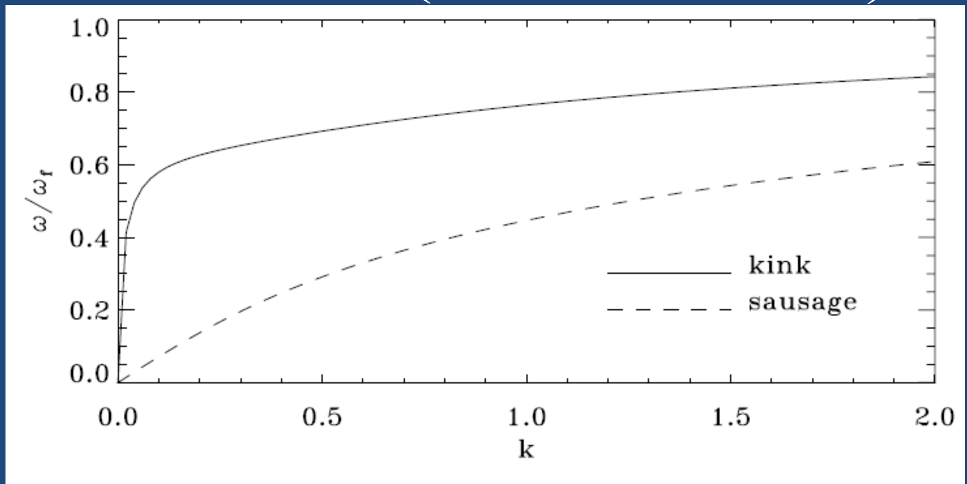
Initial configuration of the magnetic field and plasma density:

$$\rho=\cosh^{-2}(\eta z/L_{eff}),$$
$$b_{0x}=\tanh(z/L_{eff}).$$

Equation for the u_z:

$$\frac{d^2u_z}{dz^2}-2\eta\tanh(\eta z)\frac{du_z}{dz}+K^2u_z\left(\frac{\nu b_n'}{\Omega^2}\frac{\cosh^2(\eta z)}{\cosh^2(z)}-1\right)=0$$

For η=0.4 solutions are found in [Erkaev et al. 2009 AnGeo]. ⟶



We need the solution for η=1.

Generalization (Art.): B_y effect

When B_y=const is non-zero, equation for u_z takes the form

$$\frac{1}{\rho}\frac{d}{dz}\left(\rho\frac{du_z}{dz}\right)+K^2u_z\left(U(z)-1\right)=0,\quad U(z)=\frac{\nu b_n'}{\Omega^2-K^2b_m^2}\frac{1}{\rho(z)}\frac{db_{0x}(z)}{dz} \qquad (*)$$

For incompressible plasma

$$\frac{d^2u_z}{dz}+K^2u_z\left(U(z)-1\right)=0,\quad U(z)=\frac{1}{\Omega^2-K^2b_m^2}\frac{\nu b_n'}{\cosh^2(z)}$$

For the kink mode (Erkaev et al., 2009, AnGeo)

$$\Omega=\sqrt{\frac{\omega_f^2}{1+K^{-1}}},\quad \omega_f^2=\nu b_n' \quad \xrightarrow{B_y\neq 0} \quad \Omega=\sqrt{\frac{\omega_f^2}{1+K^{-1}}+b_m^2K^2}$$

Current sheet is unstable when $w_f^2<0$

$$\gamma=\sqrt{\frac{|\omega_f|^2}{1+K^{-1}}-b_m^2K^2}$$

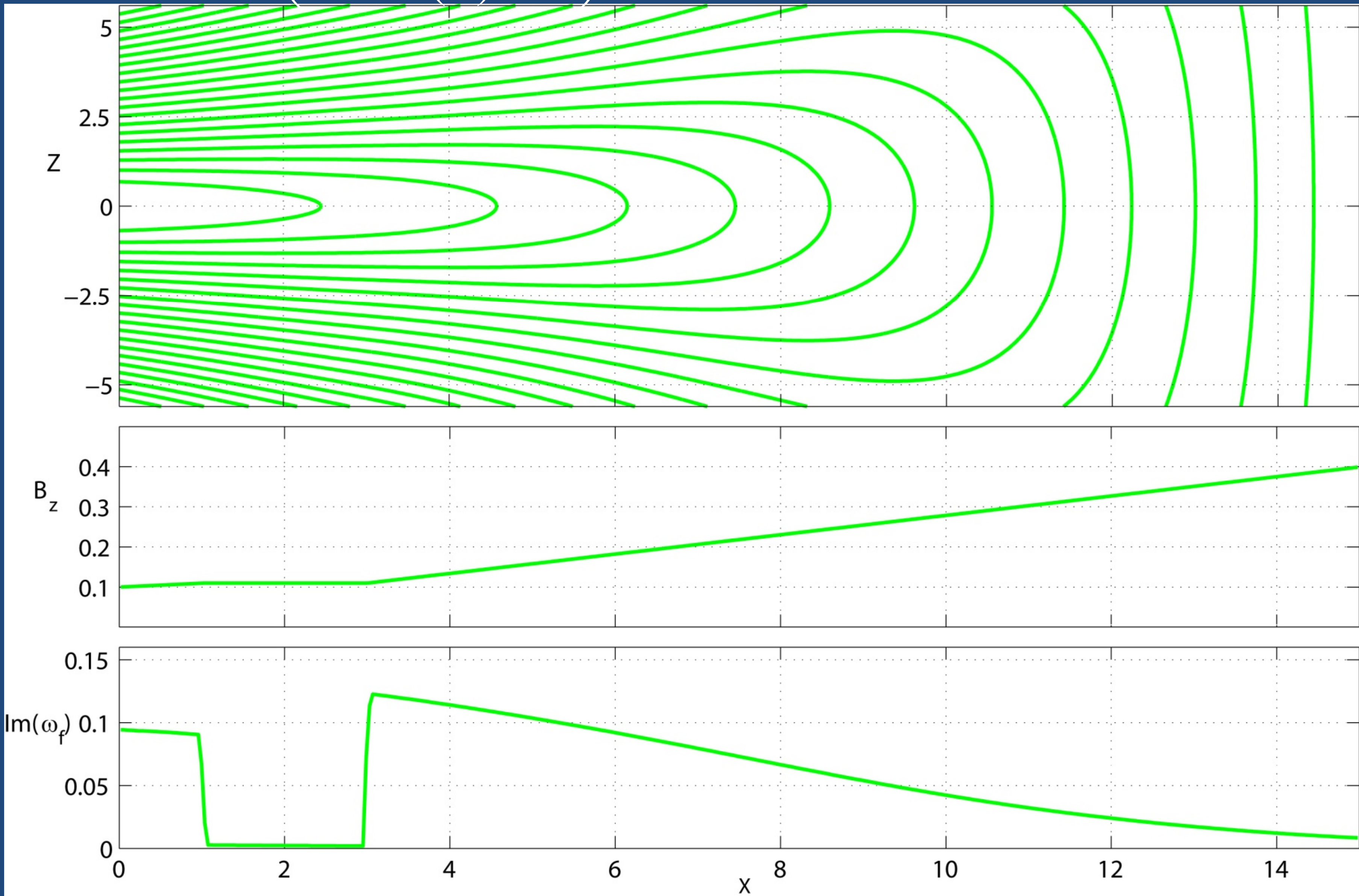
Stabilization of the short-wavelength band

For the general case of compressible plasma with η=1 solutions of Eq. (*) are obtained numerically for different values of b_m.

Background: Pritchett's solution

[*Pritchett and Coroniti, 2010, JGR*]

$$A_{0y} = \ln\left(\frac{\cosh[F(x)z]}{F(x)}\right), \qquad \rho = \frac{1}{2}\exp(-2A_{0y}) + \rho_0$$



2D MHD simulation

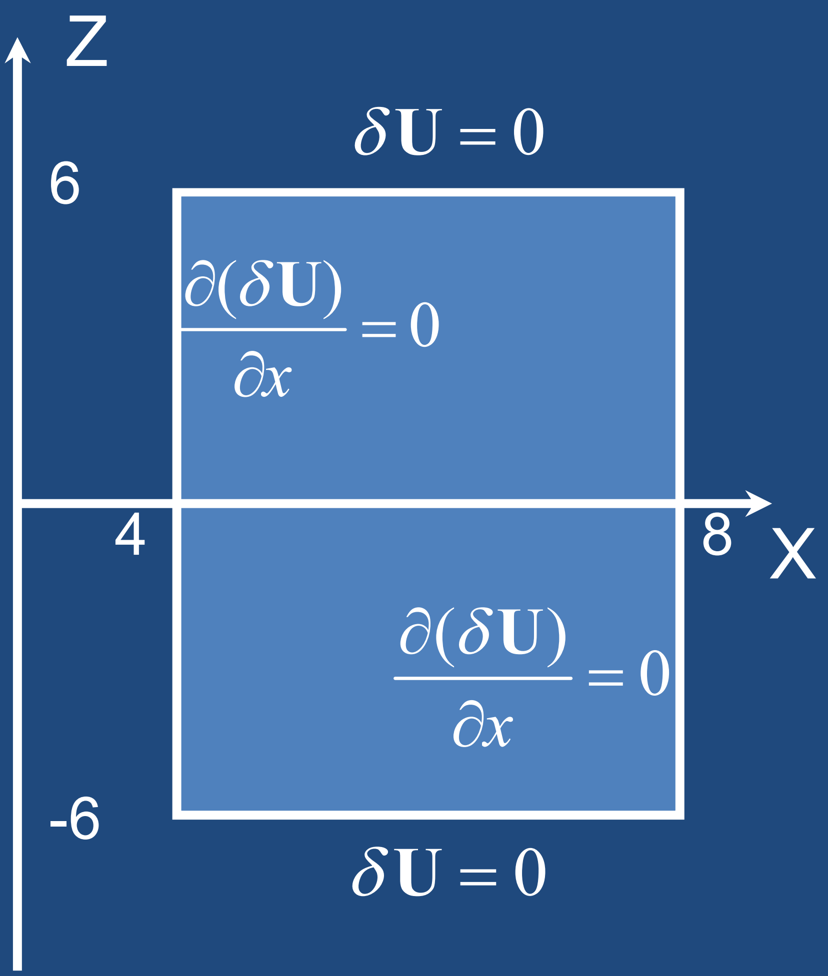
Code frame of reference:
X = -X (GSM), Y = -Y (GSM), Z = Z (GSM)

Box size
$L_x = 4$, $L_z = 12$

Resolutions
1. $N_x \times N_z = 41 \times 481$
2. $N_x \times N_z = 81 \times 961$

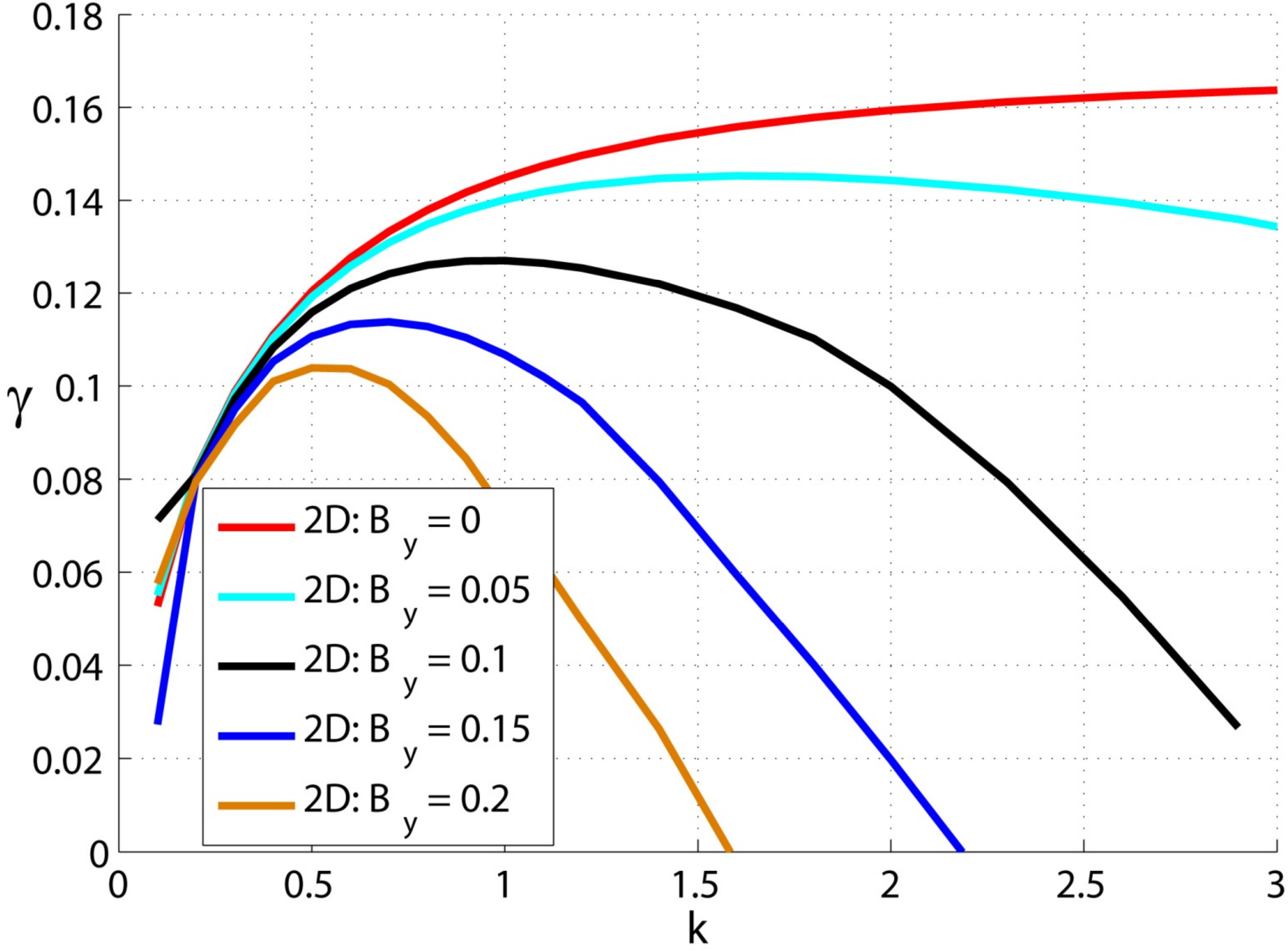
Seed perturbation
$\delta V_z = \exp(-z^2)$

Courant number
C = 0.1

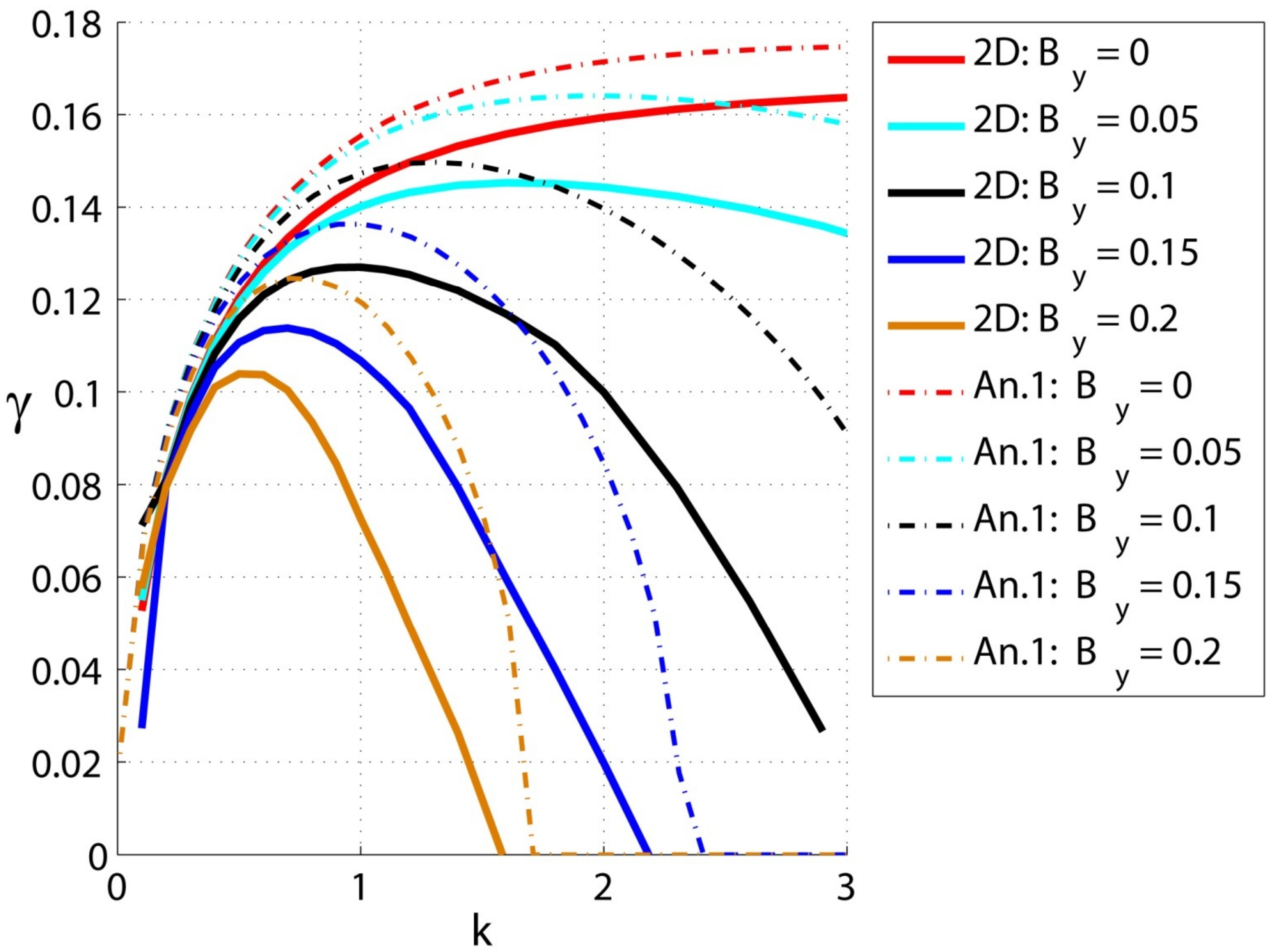


See details in [Korovinskiy et al, 2011, Adv. Sp. Res.; Korovinskiy et al., 2013, JGR]

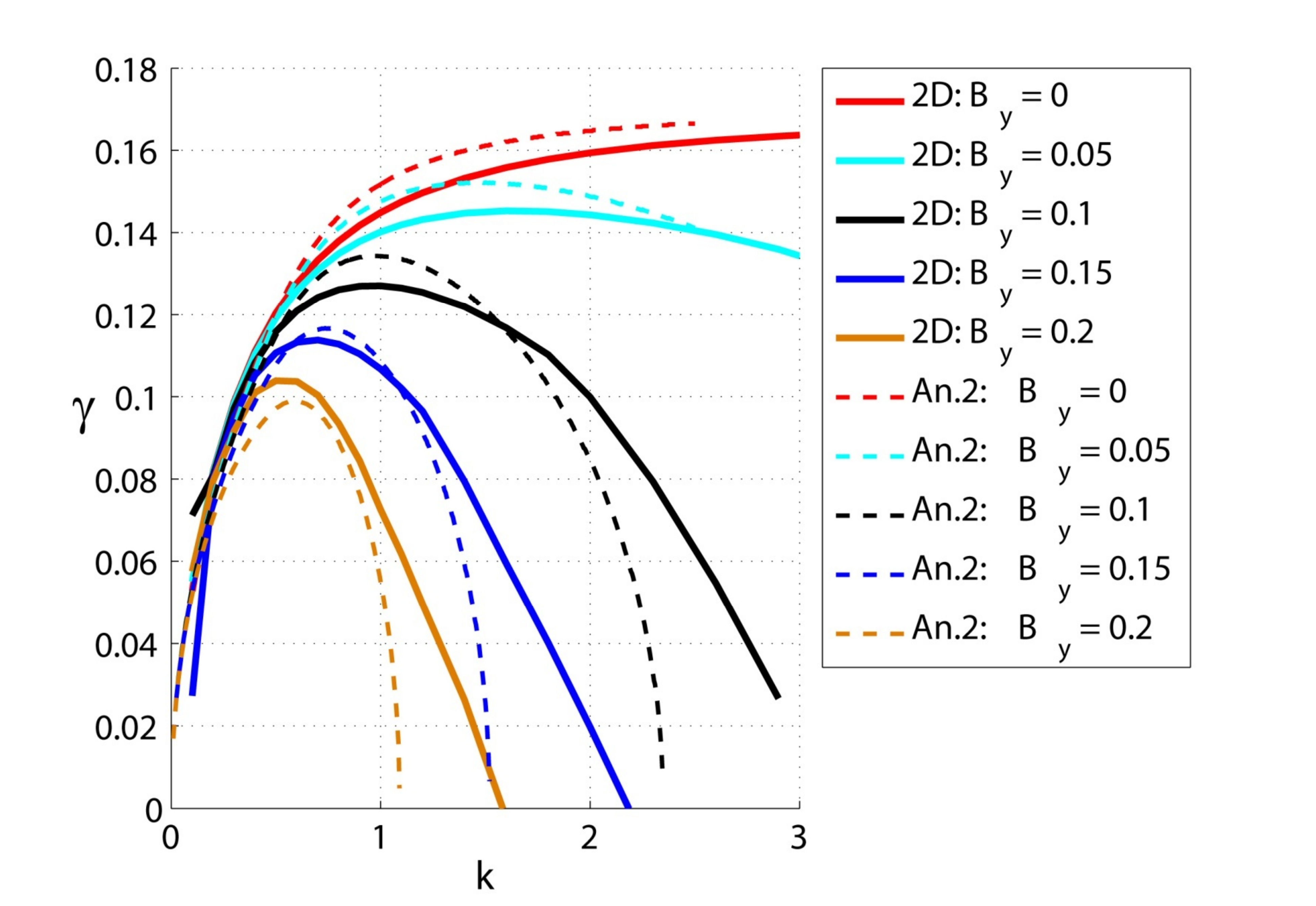
2D linearized MHD simulations



2D linearized MHD simulations + analytics (Kub.)



2D linearized MHD simulations + analytics (Art.)



3D MHD simulation

Code frame of reference:

X = –X (GSM), Y = Z (GSM), Z = Y (GSM)

Box size

$L_x = 15$, $L_y = 11.25$, $L_z = 22.5$

Resolution

$N_x \times N_y \times N_z = 384 \times 286 \times 576$

BC, relaxation procedure: [Korovinskiy et al., 2013, JGR]

BC in relaxation phase (2D in xy plane): fix the magnetic flux entering domain

$$\partial/\partial \mathbf{n}\{\rho, \mathbf{B}_\tau, p\} = 0,$$
$$\partial B_n/\partial t = 0, \quad \mathbf{V} = 0.$$

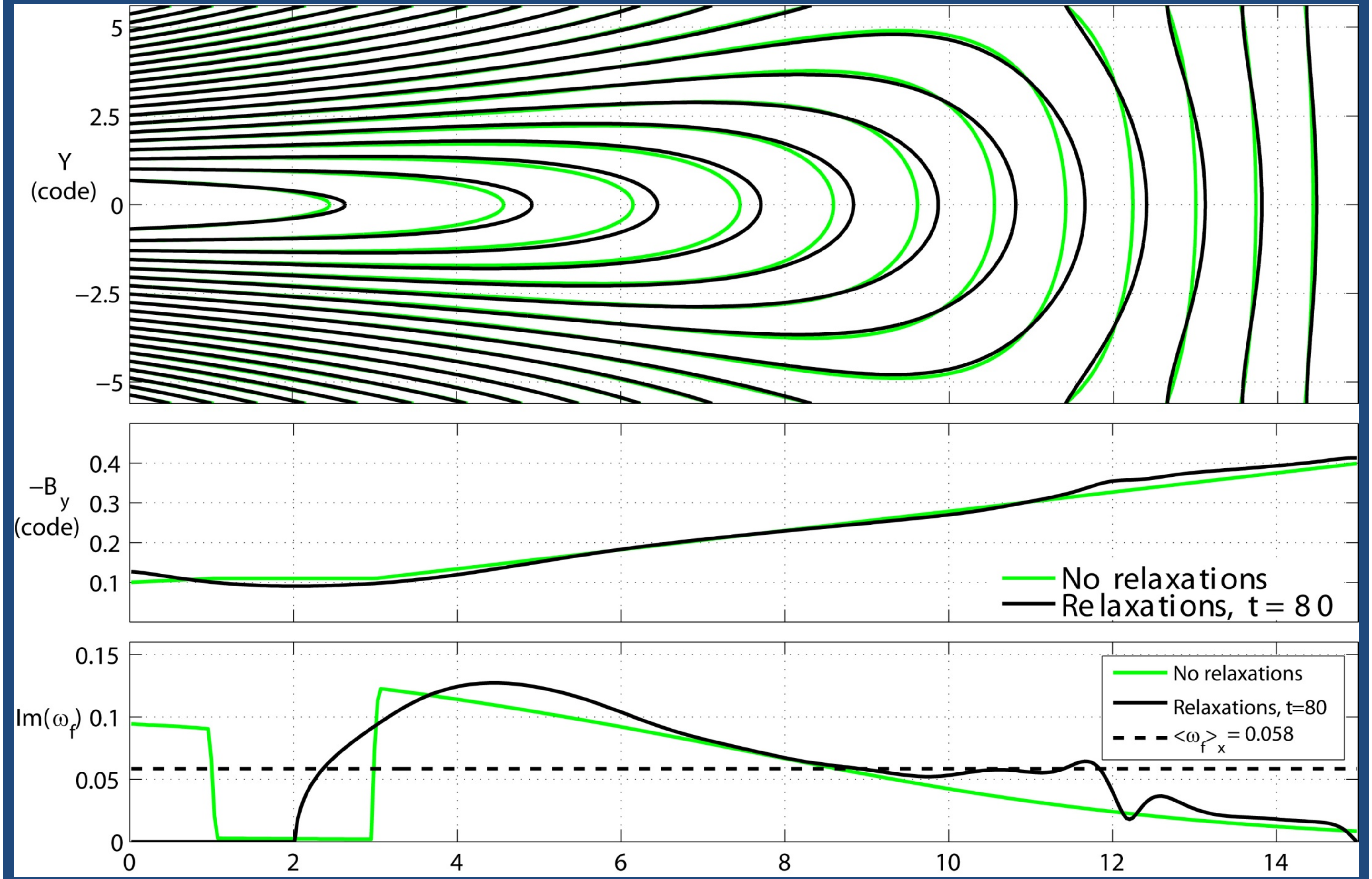
In the main phase the same BC are applied at y-boundaries, and the Earthward x-boundary

Free BC are imposed at the tailword x-boundary:

$$\partial/\partial \mathbf{n}\{\rho, \mathbf{B}, \mathbf{V}, p\} = 0$$

And at z-boundaries BC are periodic

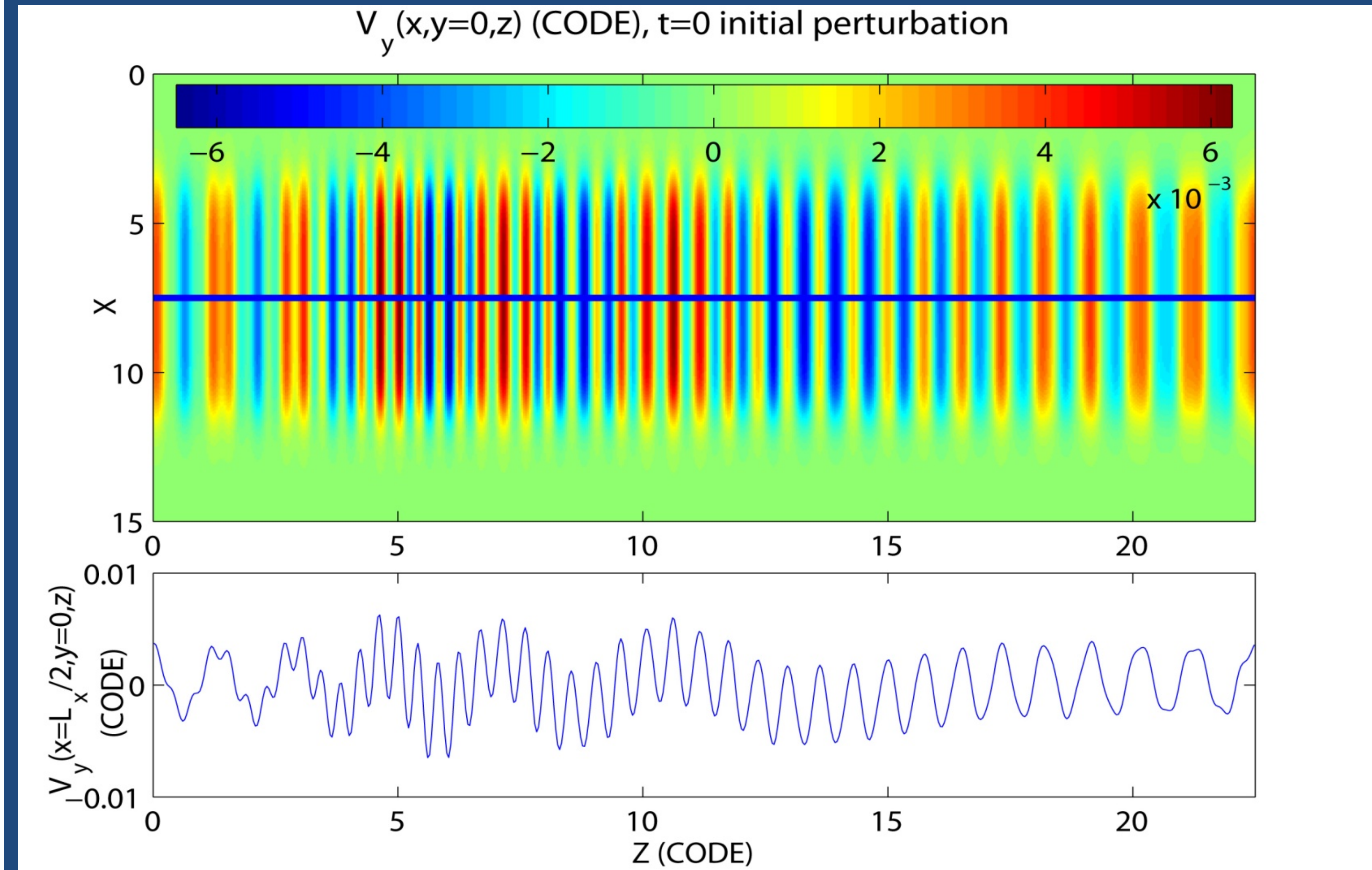
3D MHD simulation: configuration



3D MHD simulation: initial perturbation

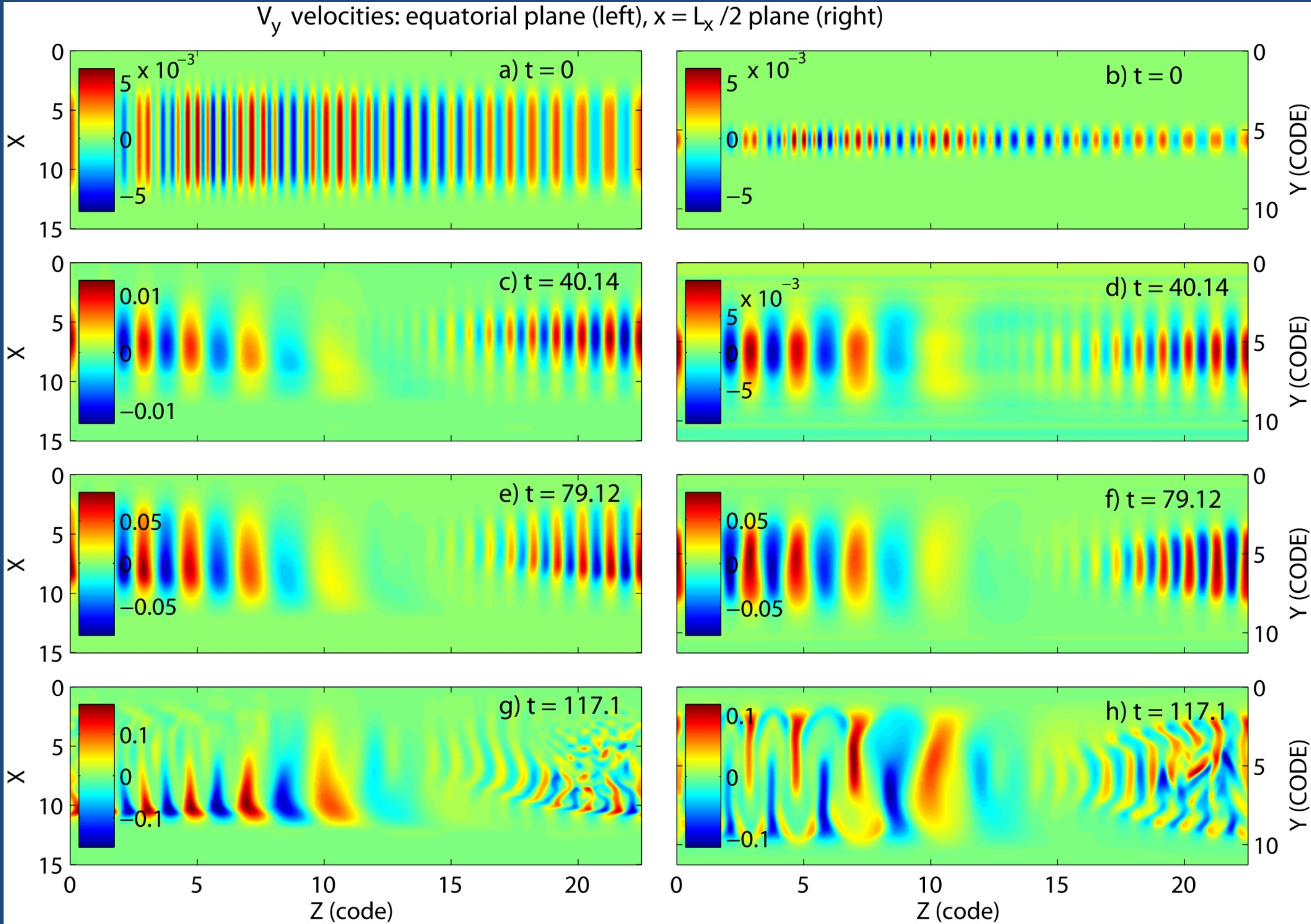
$$V_y(x,y,z,t=0) = 5\cdot 10^{-4}\sum_{m=1}^{60}\sin\left(\frac{2\pi mz}{L_z}+m^{1.5}\right)\times$$

$$\times\frac{1}{2}\left(\tanh\left(x-L_x/4\right)+\tanh\left(x-3L_x/4\right)\right)\times\exp\left(-2y^2\right)$$

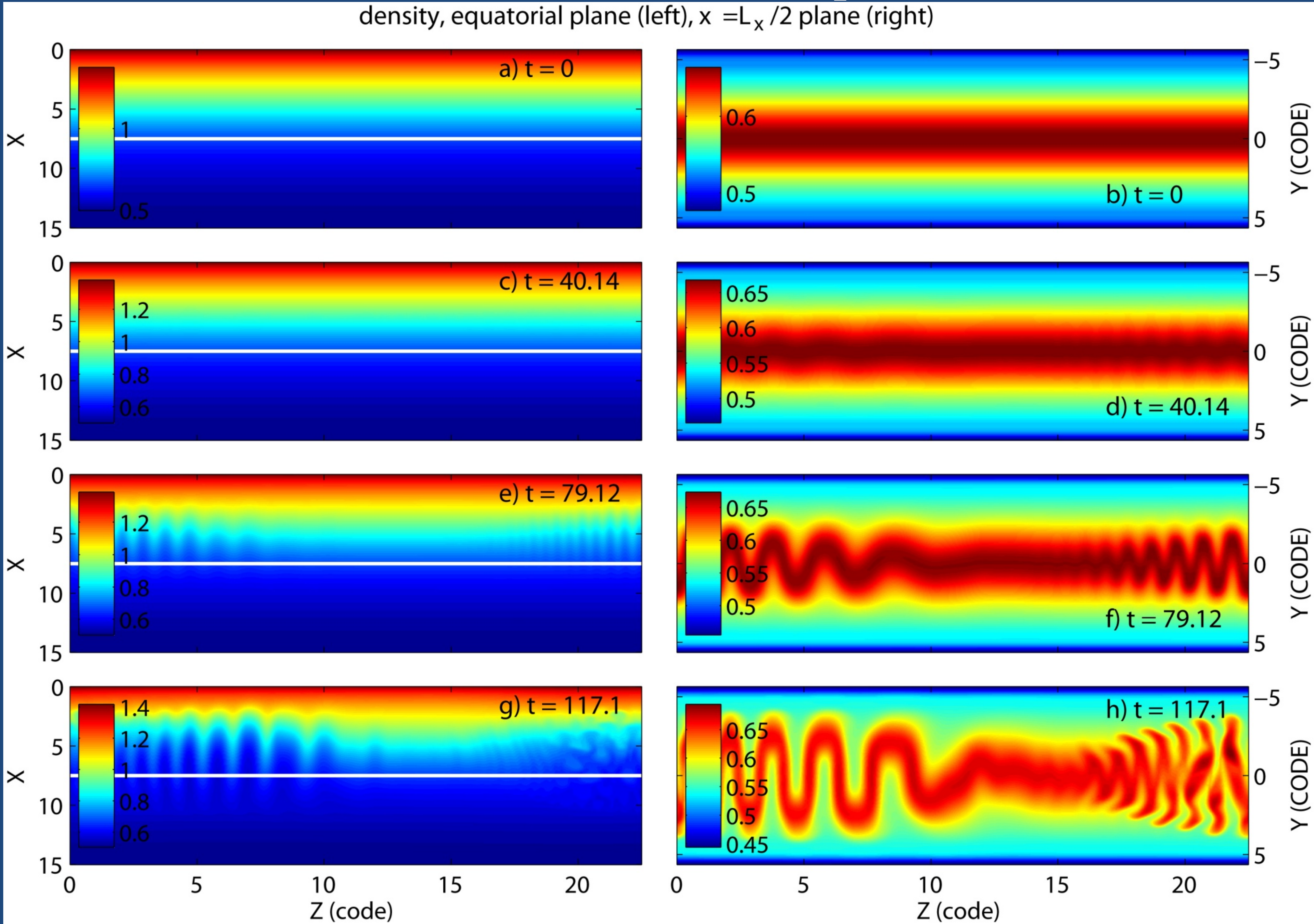


Wavenumbers $k_z = 2\pi m/L_z$ covered are $0.28 < k_z < 16.8$

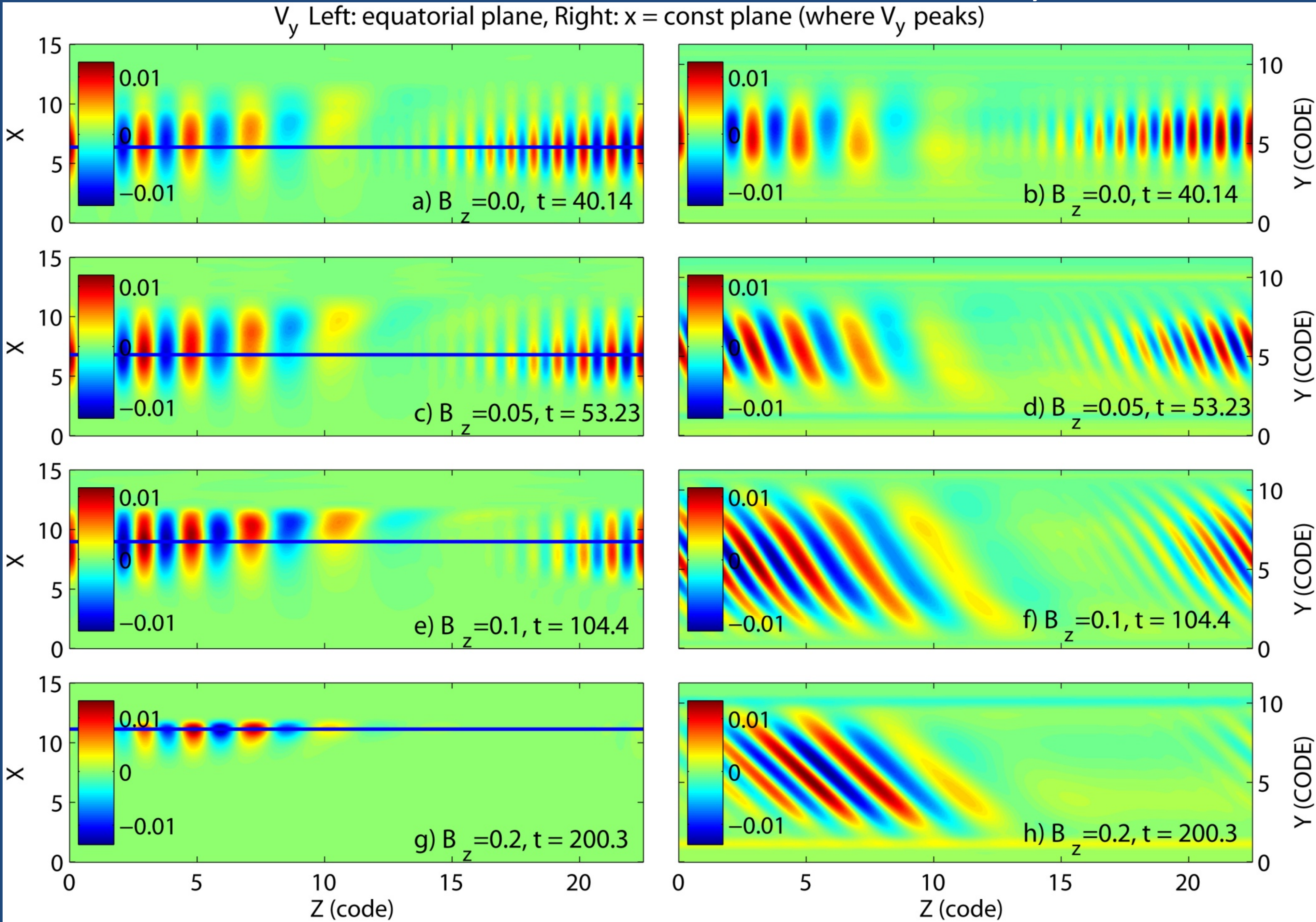
3D MHD simulation: $B_z = 0$, $V_y(t)$



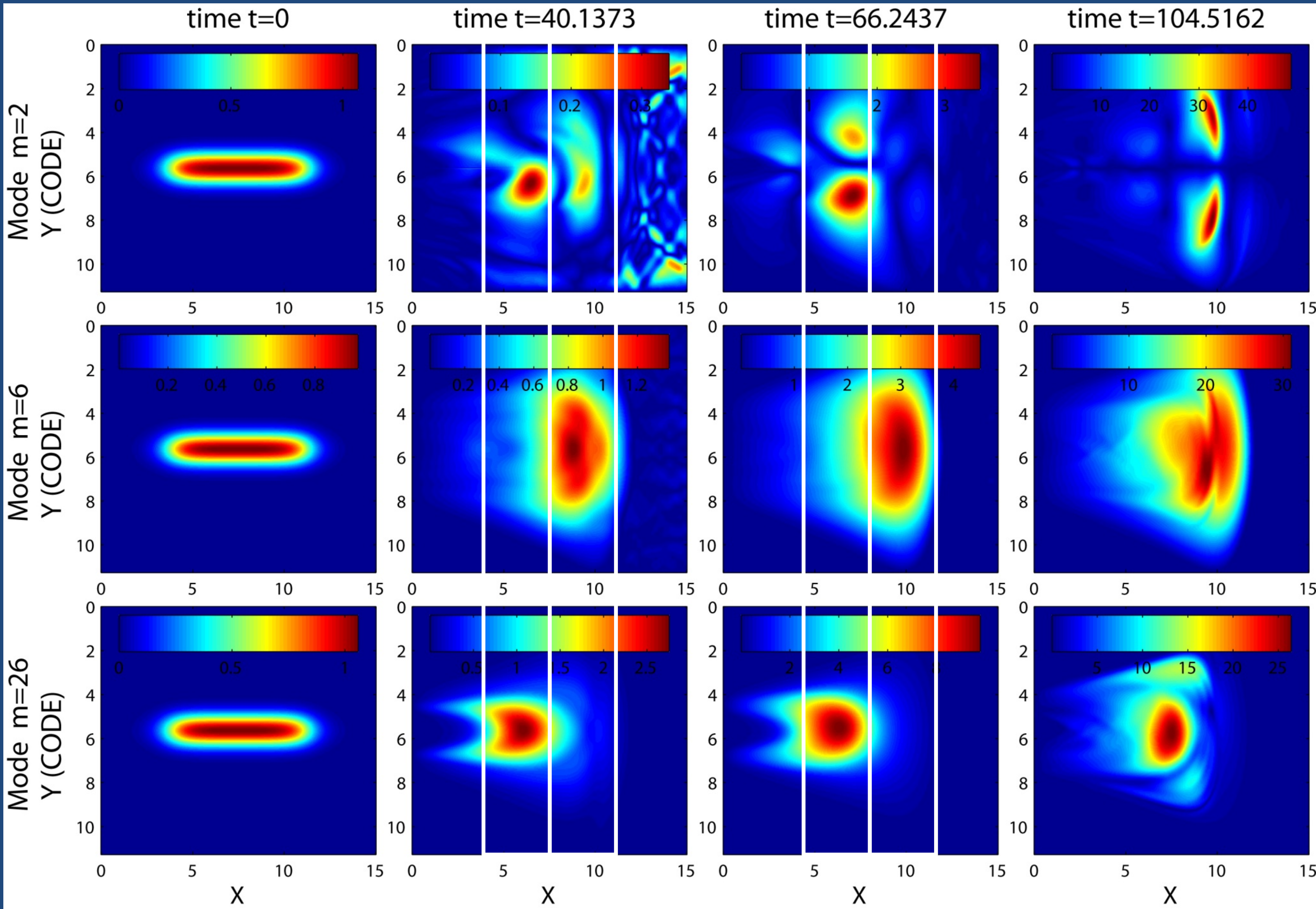
3D MHD simulation: $B_z = 0$, $\rho(t)$



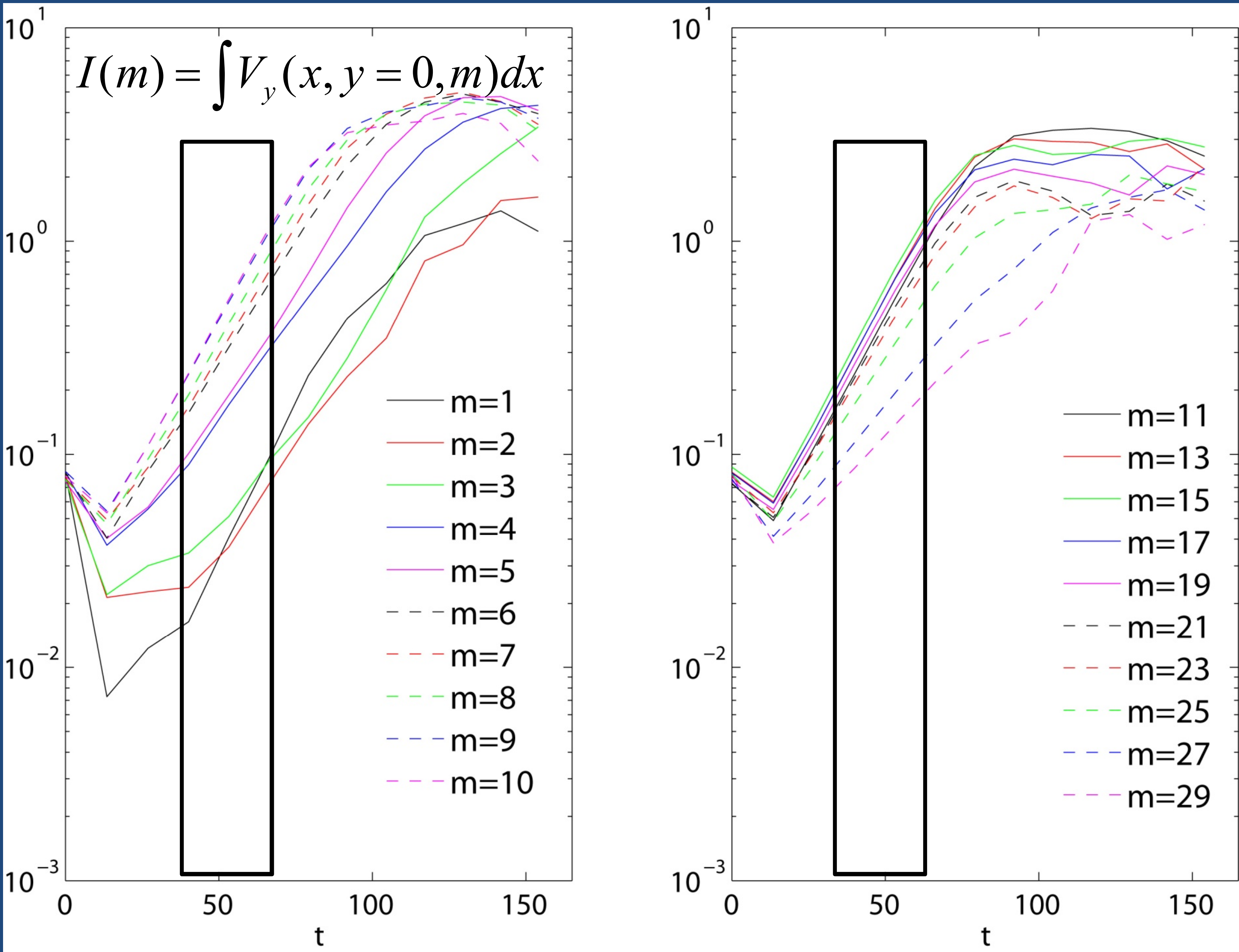
3D MHD simulation: B_z varies, $V_y(t)$



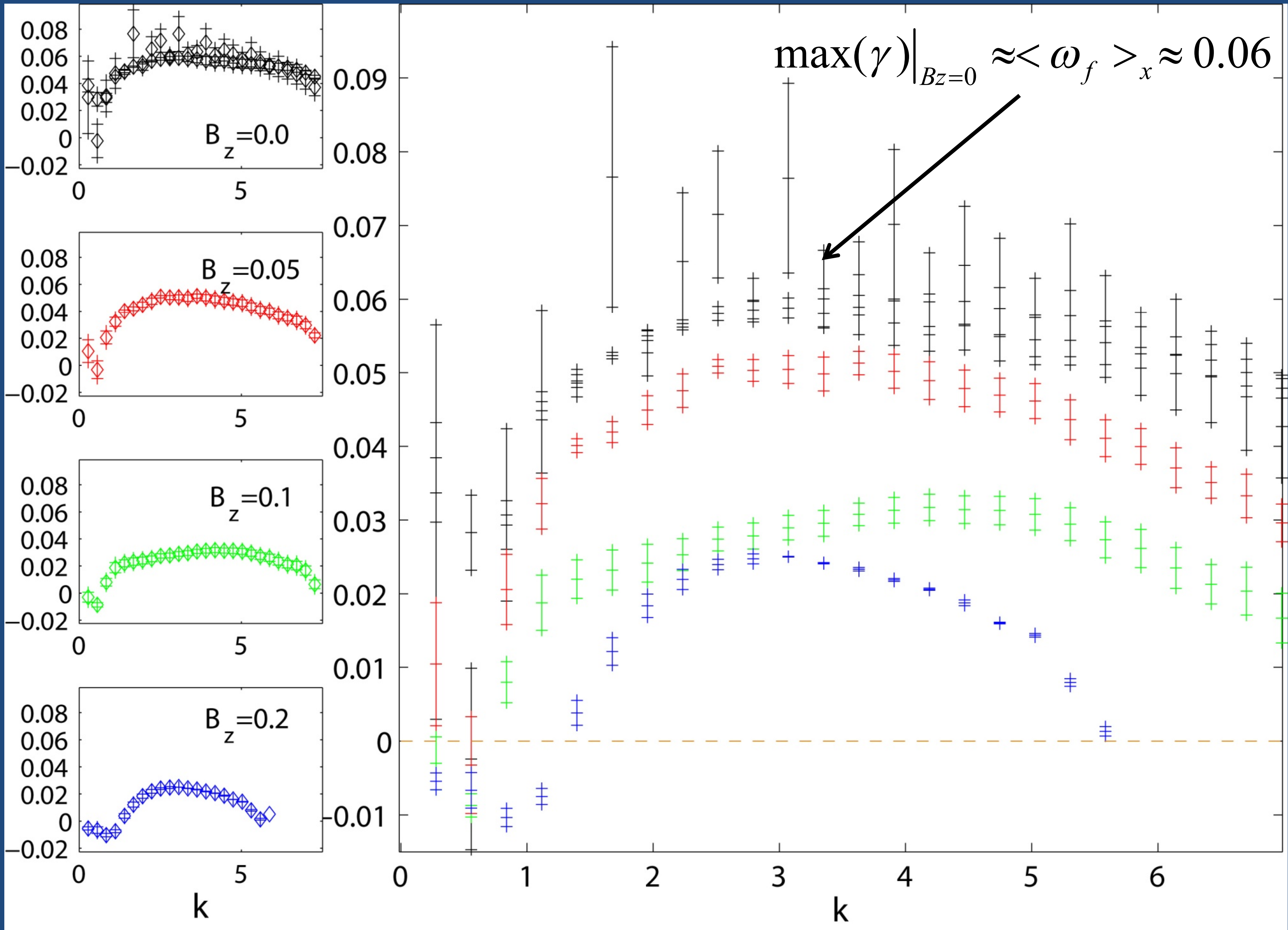
3D MHD simulation: $B_z = 0$, $V_y(x,y,m)$



3D MHD simulation: B_z =0, I(m)



3D MHD simulation: growth rates



Conclusions

1. Наличие ненулевой By компоненты уменьшает инкремент неустойчивости, подавляя коротковолновые возмущения;
2. Аналитические выражения и двухмерное МГД моделирование дают близкие оценки для дисперсионной кривой;
3. В трехмерном моделировании картина качественно та же, но затухание неустойчивости происходит при больших (примерно в 4 раза) значениях волнового вектора;
4. **Дисперсионная кривая дабл-градиент неустойчивости демонстрирует наличие максимума даже при нулевом By;**
5. **Максимальное значение инкремента неустойчивости очень близко к усредненной по слою аналитической оценке (совпадает с результатом в [Korovinskiy et al., 2013, JGR]);**
6. **Ненулевая компонента By приводит к поляризации возмущений в плоскости YZ.**