ФОРМИРОВАНИЕ УСКОРЕННЫХ ХВОСТОВ РАСПРЕДЕЛЕНИЯ ЭЛЕКТРОНОВ В ПРОЦЕССЕ НАГРЕВА

И.Ф. Потапенко
ИПМ им. М.В. Келдыша РАН

С.А. Карпов
ВНИИА им. Н.Л. Духова

Spatial temporal scaling

$$\frac{\text{mean free path} \quad \lambda}{\text{characteristic size of the object} \quad L}$$

Kn - number

$\lambda / L: \quad \ll 1 \quad - MHD,$

$\lambda / L: \quad \gg 1 \quad - Vlasov\ eq.,$

$\lambda / L: \quad \sim 1 \quad - LFP\ eq.$

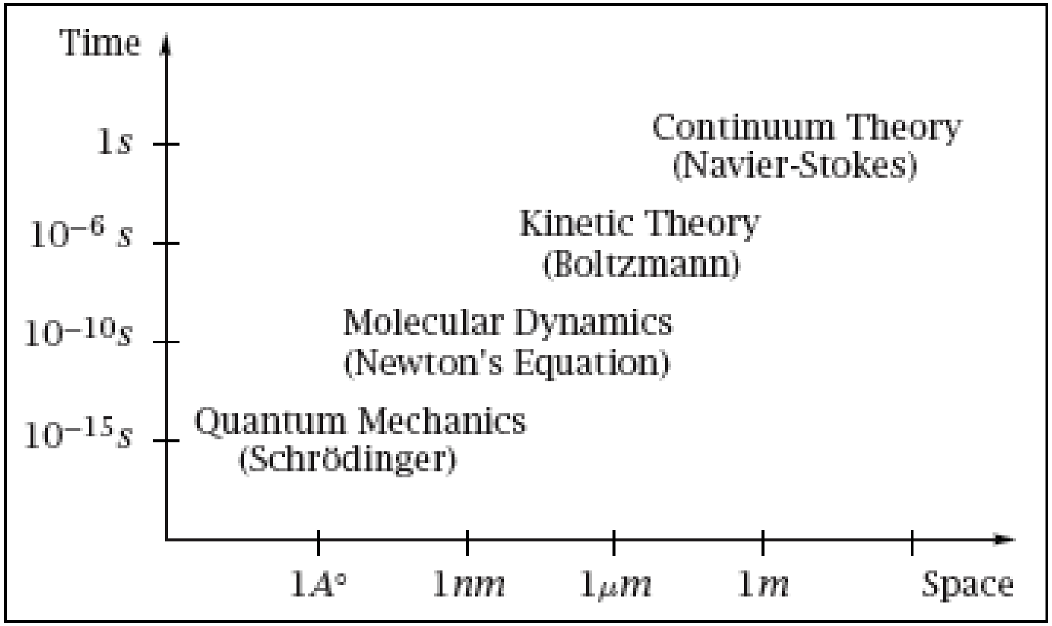


Figure 1. Different laws of physics are required to describe properties and processes of fluids at different scales.

Figure: from E & Engquist, AMS Notice

Kinetic description of plasmas: Vlasov eq. + LFP eq.

Weakly collisional plasma

Kinetic description of plasmas:

Vlasov equation + **LFP equation**

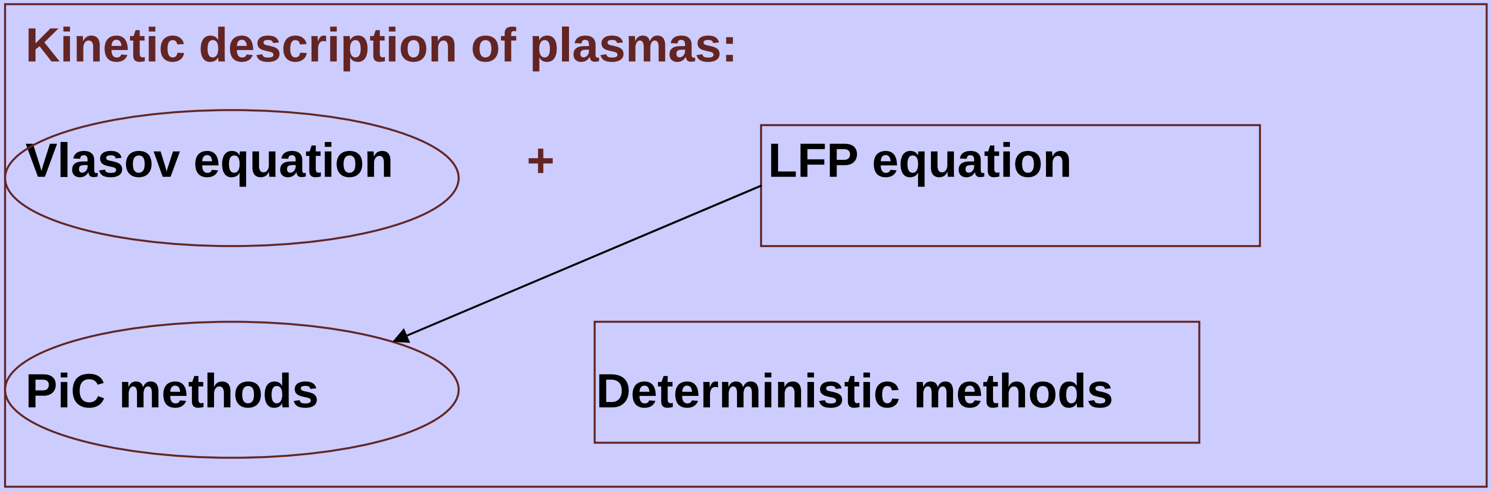
PiC methods **Deterministic methods**

i-n------

e-e, e-i,

1D3V

Analytical asymptotic results are confirmed with high accuracy by the numerical computing of the non-linear kinetic equation (deterministic and stochastic) and vice versa.



e-e, e-i,
1D3V

Бобылев А.В., Потапенко И.Ф., Карпов С.А. Метод. Монте-Карло для двухкомпонентной плазмы/ Мат. Моделирование, 2012

A.V. Bobylev and I.F. Potapenko **A general approach to Monte Carlo methods for Coulomb collisions** *J. Comp. Phys.*, 2013

Бобылев А.В., Потапенко И.Ф., Карпов С.А. Моделирование методом Монте-Карло кинетического столкновительного уравнения с внешними полями, Мат. моделирование, 2014

Препринты ИПМ 2012, 2013 – на сайте ИПМ им. М.В.Келдыша

OUTLINE

Boltzmann equation $\longrightarrow$ **Landau-Fokker-Planck equation**

DSMC simulation $\longleftrightarrow$ **Landau-Fokker-Planck equation**

Examples. $U \sim 1/r^{\beta}$, $1 \le \beta \le 4$;

Influence of heating on the temporal plasma relaxation on the base of the kinetic equation with the nonlinear Landau-Fokker-Planck *collisional operator.*
Heating terms: an electrical field, a quasi-linear diffusion operator

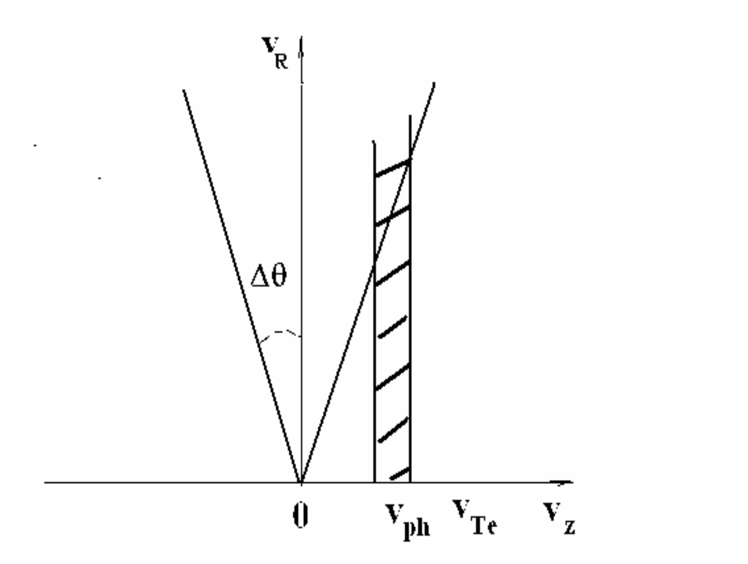
Runaway particles for grazing interaction - $E(f)$

Self-similar solutions- $D(f)$

(aim → algorithm verification & validation)

$$\frac{\partial f}{\partial t} = C(f,f) + H(f); \quad 0 \le v < \infty, t \ge 0, \qquad H(f) \in \begin{matrix}[v_1, v_2] \\ [0,\infty)\end{matrix}$$

$$H(f) \rightarrow \left\{ \begin{aligned} & D(f) = \frac{1}{v^2}\frac{\partial}{\partial v}\left[v^2 D(v,t)\frac{\partial f}{\partial v} \right] \\ & E(f) = \gamma \frac{\partial f}{\partial v} \end{aligned} \right\}$$



$$ВЧ \quad нагрев, \quad v_{ph} \pm \Delta v_{ph}. \quad v_{ph}(t) \approx \Omega / k_{\|}$$

Потапенко И.Ф. Формирование функции распределения электронов в слабостолкновительной плазме в процессе нагрева. //Инженерная физика. № 5, 2012, С. 31

Potapenko I. F., Krasheninnikov S. I. Numerical solution of nonlinear electron kinetic equation in self-similar variables // J. Plasma Phys. v. 77 n 6, 2011. P. 803-812

Kinetic description of plasmas

$f_\alpha(\mathbf{v},\mathbf{r},\mathrm{t})$ - **distribution function**

$$\int_{\mathbf{R}_3} f_\alpha(\mathbf{v},\mathbf{r},\mathrm{t})\mathrm{d}^3\mathbf{v} = n_\alpha(\mathbf{r},\mathrm{t})$$

<u>Kinetic equation:</u>

$$\boxed{D_\alpha f_\alpha = \sum_\beta Q_{\alpha\beta}[f_\alpha, f_\beta]}$$

$$D_\alpha f_\alpha = \left\{\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} + \frac{\mathbf{F}}{m}\frac{\partial}{\partial \mathbf{v}}\right\} f_\alpha$$

<u>Boltzmann collision integral</u>

$$\boxed{Q_{\alpha\beta}(f_\alpha, f_\beta) = \int_{\mathbb{R}^3\times\mathbf{S}^2} \mathrm{d}\mathbf{w}\,\mathrm{d}\mathbf{n}\; g_{\alpha\beta}(\mathrm{u},\mu)\left\{f_\alpha(\mathbf{v}')f_\beta(\mathbf{w}') - f_\alpha(\mathbf{v})f_\beta(\mathbf{w})\right\}}$$

$$g_{\alpha\beta}(\mathrm{u},\mu) = \mathrm{u}\cdot\sigma_{\alpha\beta}(\mathrm{u},\mu)$$

$$\mathbf{v}' = \mathbf{U} + \frac{m_{\alpha\beta}}{m_\alpha}\mathrm{u}\,\mathbf{n}, \qquad \mathbf{U} = \frac{m_\alpha\mathbf{v} + m_\beta\mathbf{w}}{m_\alpha + m_\beta}$$

$$\mathrm{u} = |\mathbf{v} - \mathbf{w}|, \quad \mu = \frac{\mathbf{u}\cdot\mathbf{n}}{\mathrm{u}}$$

$$\mathbf{w}' = \mathbf{U} - \frac{m_{\alpha\beta}}{m_\beta}\mathrm{u}\,\mathbf{n}, \qquad m_{\alpha\beta} = \frac{m_\alpha m_\beta}{m_\alpha + m_\beta}$$

$U \sim 1/r$

Landau-Fokker-Planck integral – Landau (1936), Rosenbluth et al (1957)

$$\boxed{Q_{\alpha\beta}^{(L)}(f_\alpha, f_\beta) = \frac{m_{\alpha\beta}^2}{2m_\alpha^2}\frac{\partial}{\partial \mathrm{v}_i}\int d^3\mathbf{w}\ \mathrm{u}\,\sigma_{\alpha\beta}^{(tr)}(\mathrm{u}) R_{ij}(\mathbf{u})\left\{\frac{\partial}{\partial \mathrm{v}_j} - \frac{m_\alpha}{m_\beta}\frac{\partial}{\partial \mathrm{w}_j}\right\} f_\alpha(\mathbf{v}) f_\beta(\mathbf{w})}$$

$\mathrm{u} = |\mathbf{v} - \mathbf{w}|$ $\quad R_{ij}(\mathbf{u}) = \mathrm{u}^2\delta_{ij} - \mathrm{u}_i \cdot \mathrm{u}_j$ $\quad g_{\alpha\beta}^{(tr)} = u\,\sigma_{\alpha\beta}^{(tr)}(\mathrm{u}) = 4\pi u\left(\frac{e_\alpha e_\beta}{m_{\alpha\beta}\mathrm{u}^2}\right)^2 L_{\alpha\beta}$

grazing collisons

Numerical solution:

Splitting over physical parameters

Vlasov equation (PIC)

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}\frac{\partial f_\alpha}{\partial \mathbf{r}} + \frac{e_\alpha}{m_\alpha}\left(\mathbf{E}_\mathrm{s} + \frac{1}{c}\mathbf{v}\times\mathbf{B}_\mathrm{s}\right)\frac{\partial f_\alpha}{\partial \mathbf{v}} = 0$$

Landau_Fokker-Planck equation

$$\frac{\partial f_\alpha}{\partial t} + \frac{e_\alpha}{m_\alpha}\left(\mathbf{E}_\mathrm{ex} + \frac{1}{c}\mathbf{v}\times\mathbf{B}_\mathrm{ex}\right)\frac{\partial f_\alpha}{\partial \mathbf{v}} = \sum_\beta Q_{\alpha\beta}^{(L)}(f_\alpha, f_\beta)$$

$\mathbf{E}_\mathrm{s}(\mathbf{r},t), \mathbf{B}_\mathrm{s}(\mathbf{r},t)$ – self-consistent and external fields

$\mathbf{E}_\mathrm{ex}(t), \mathbf{B}_\mathrm{ex}(t)$ – external fields (spatially uniform frame)

LFP equation and DSMC simulation

LFP equation and DSMC simulation

LFP equation

↓

Boltzmann equation

↓

DSMC simulation

model cross section:

$$\boxed{g_{\alpha\beta}(\mathrm{u},\mu,\varepsilon)=\frac{1}{2\pi\varepsilon}\delta\left[(1-\mu)-\varepsilon a_{\alpha\beta}(\mathrm{u})\right]}$$

$$a_{\alpha\beta}(u)=\begin{cases} g_{\alpha\beta}^{(tr)}(u), \text{ for} & 0<\varepsilon g_{\alpha\beta}^{(tr)}\leq 2 \\ 2/\varepsilon, & \text{otherwise}\end{cases}$$

<u>Important consequences:</u>

$$\mu=\cos\theta=1-Min\left\{\frac{a\varepsilon}{\mathrm{u}^3},2\right\},\ a=const$$

$$g_{\alpha\beta}^{tot}(u,\varepsilon)=2\pi\int_{-1}^{1}d\mu\, g_{\alpha\beta}\left(u,\mu,\varepsilon\right)=\varepsilon^{-1}=const$$

<u>The collision frequency of particles of any kind is constant</u> – “quasi Maxwellian”

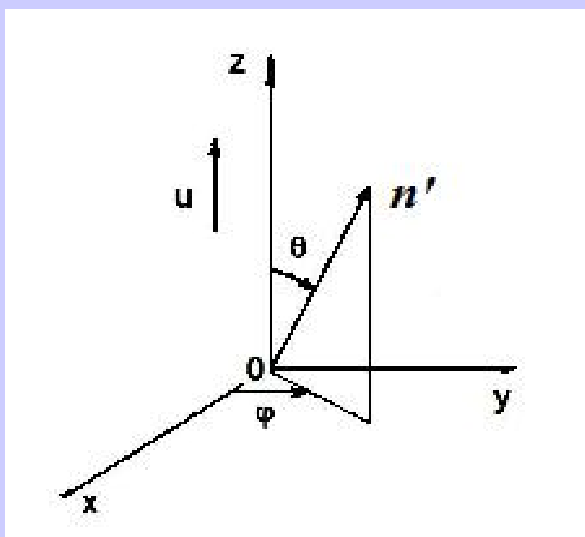
a
l
g
o
r
i
t
h
m

$$V_N(t)=\{v_1(t),v_2(t),...,v_N(t)\}\in R_{3N} \rightarrow \left(\sum_{1\le i\le j\le n} p_{ij}=1,\right)\rightarrow \{v_i^{(\alpha)},v_j^{(\beta)}\}\rightarrow$$

$$v'_i=\frac{1}{m_\alpha+m_\beta}(m_\alpha v_i+m_\beta v_j+m_\beta u n'), \quad v'_j=\frac{1}{m_\alpha+m_\beta}(m_\alpha v_i+m_\beta v_j-m_\alpha u n'); \rightarrow$$

$$\mu=\cos\theta=1-Min\left\{\frac{a\varepsilon}{u^3},2\right\}, \; a=const; \quad n'=\frac{u'}{|u|}, \quad \varphi=2\pi r, \; r\in[0,1); \quad \rightarrow$$

$\tau_N-\varepsilon/N;$ $t=t+\tau_N;$ *after N steps:* $V_N=V_N(t+\Delta t)$



All conservation laws

N operations

3V

Simulation parameters for stochastic DSMC modeling :

N - number of particles;

ε - approximation parameter;

M - number of simulation runs;

$\rho = m_e / m_i$ - mass ration;

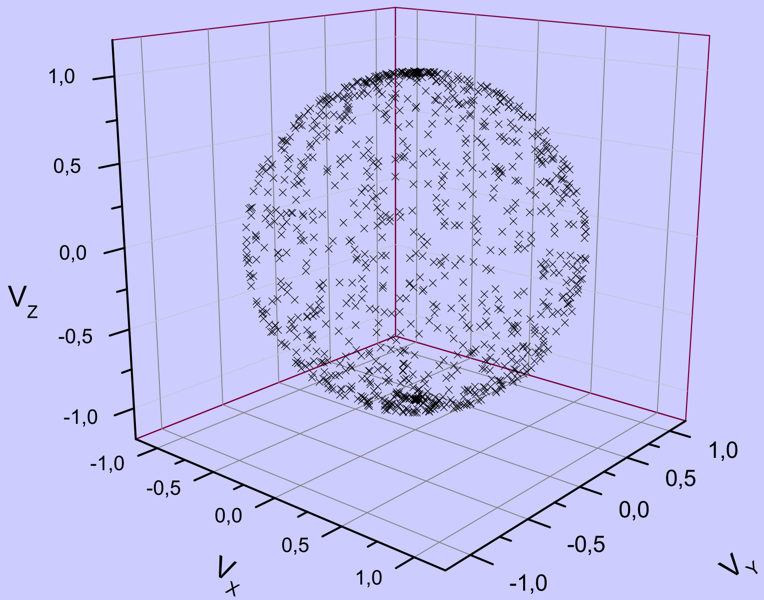
$\xi = \left(\rho \cdot T_i / T_e\right)^{1/2}$ - ratio of the initial thermal velocities.

What we can compute? - Distribution function moments:

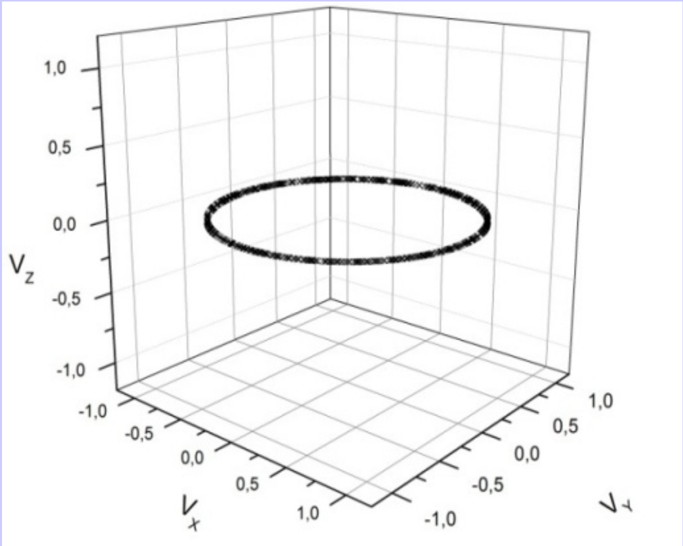
$$m_{2n}(t) = \int_{\mathbf{R}_3} v^{2n} f(\mathbf{v},t)\,\mathrm{d}\,\mathbf{v} \;\Rightarrow\; f(\mathbf{v},t) = \frac{1}{N}\cdot\sum_{j=1}^{N}\delta(\mathbf{v}-\mathbf{v}_j) \Rightarrow M_{2n}(t) = \frac{1}{N}\sum_{j=1}^{N}\left|\mathbf{v}_j\right|^{2n}$$

$$m_1 \;- current,\; m_2 \;- energy, \qquad \delta(|\,\mathbf{v}\,|-1): \quad m(0)=1, \quad m(t) \underset{t\to\infty}{\rightarrow} m_{Maxw}$$

Initial distribution function - $f_0 = \delta(|v|-1)$



$$f_{e,i}(v,\mu) = \frac{1}{v^2}\delta(v - v_{e,i})\delta(\mu)$$



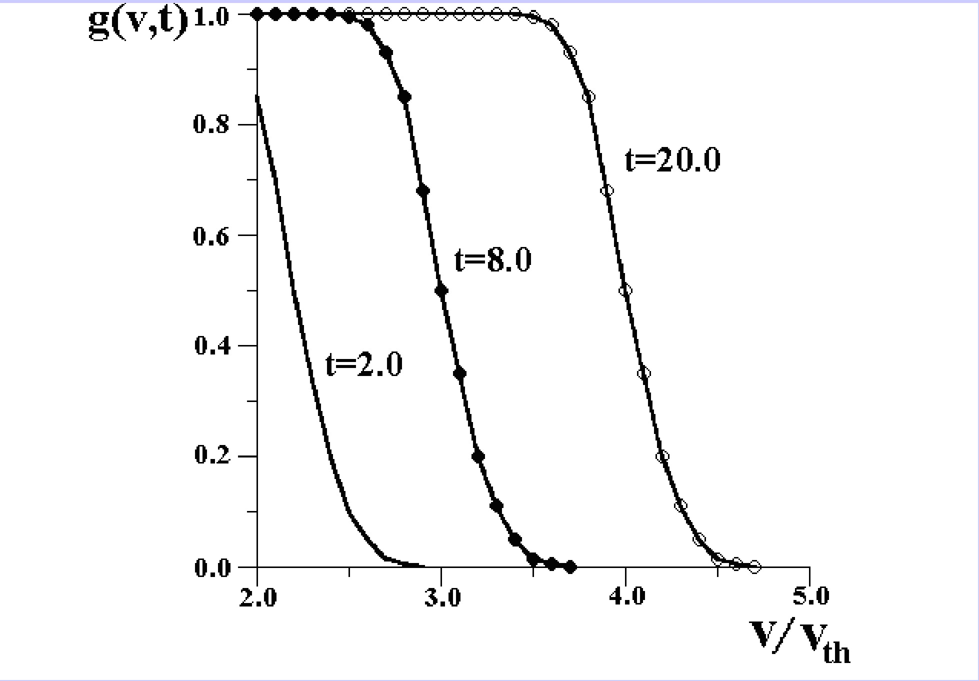
New DSMC method is compared with results obtained by completely conservative finite difference schemes

$$U - 1/r$$

$$f \approx f_{Maxw} \cdot e^{-\xi^{5/2}}$$

$$g(v,t) = f / f_{Maxw},$$

$$Tail\ relaxation\ time: t_f = t_0 \frac{v_f^3}{v_{th}^3}$$



Front velocity

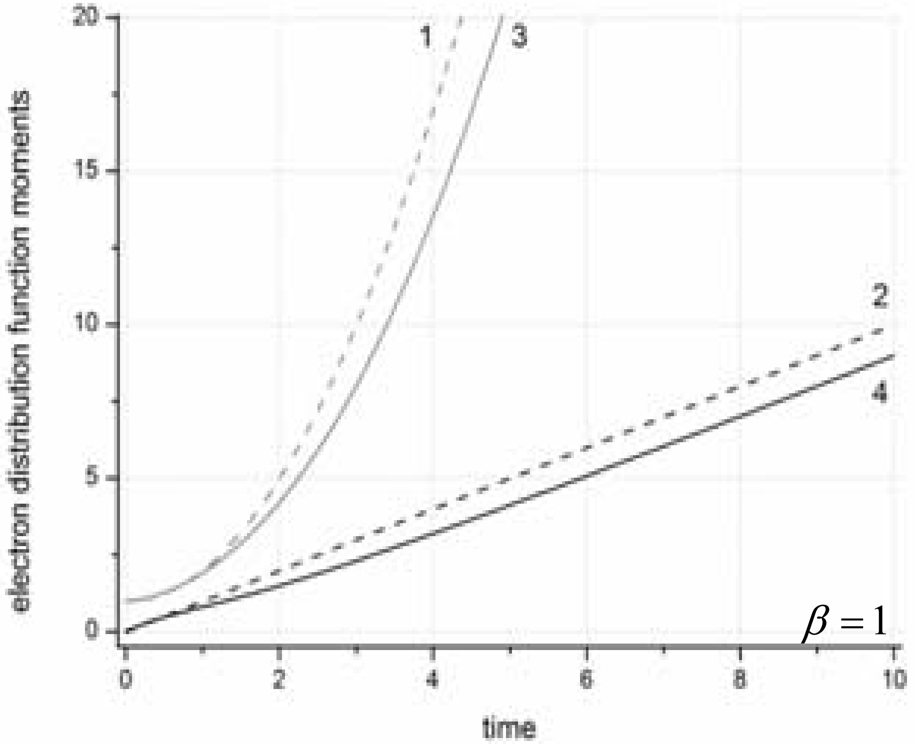
$$\frac{dv_f}{dt} \approx \frac{1}{v_f^2}, \quad v_f \approx (3t)^{1/3}$$

Width of the front $\Delta_f(t) = \sqrt{\pi}$

Plateau: $v_1 \le D_{\parallel} f \le v_2$

Ranaway “electrons” in two-component spatially uniform plasmas

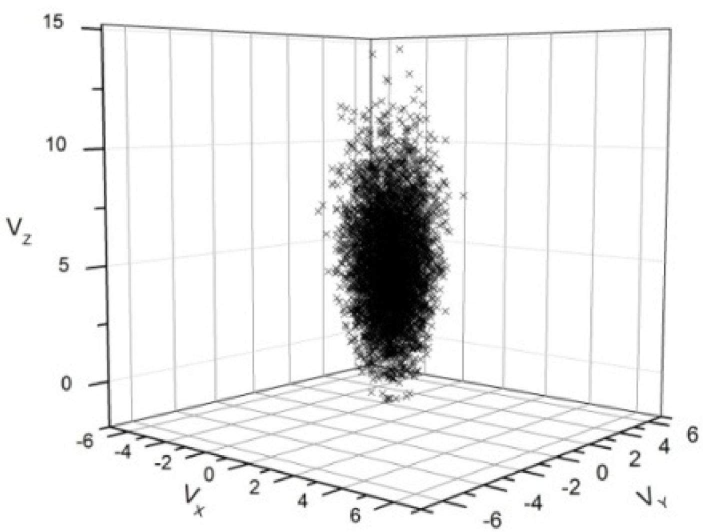
$$\frac{\partial f_e}{\partial t}+\frac{e_\alpha E_z}{m_\alpha}\frac{\partial f_e}{\partial v_z}=Q_{ee}+Q_{ei}$$



$$\frac{d\mathbf{v}_\alpha}{dt}=\frac{e_\alpha \mathbf{E}}{m_\alpha}, \qquad \gamma=\frac{e_\alpha \mathbf{E}}{m_\alpha} \quad \Rightarrow$$

$$\mathbf{v}_i(t+\Delta t)=\mathbf{v}_i(t)+\gamma\,\Delta t$$

$$U\sim 1/r,$$



$$N=1000, M=20, \varepsilon=0.01$$

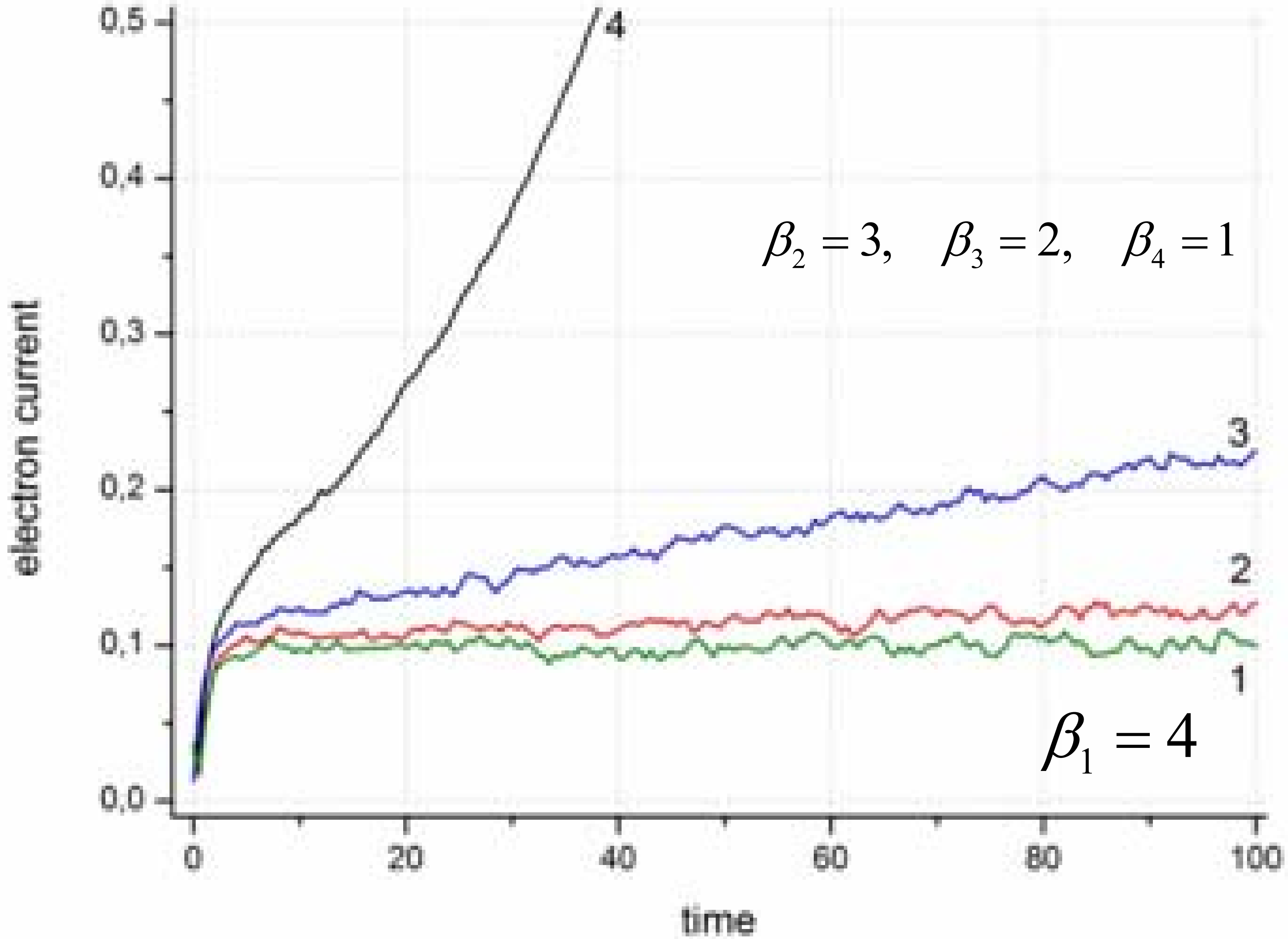
$$\rho=1/1800,\ \gamma=0.1$$

$U \sim 1/r^{\beta}, \quad 1 \le \beta \le 4;$

$\beta = 1$ – Coulomb interaction

$\beta = 2$ – dipole interaction

$\beta = 4$ – Maxwellian molecules



$\beta_2 = 3, \quad \beta_3 = 2, \quad \beta_4 = 1$

$\beta_1 = 4$

Self-Similar Solutions:

$$\frac{\partial f}{\partial t} = C(f,f) + H(f), \qquad \xi = \frac{mv^2}{2T}$$

- *Asymptotic results*
- enhanced tails

$$H(f) \quad \to \quad D(f) = \frac{1}{\xi^{1/2}} \frac{\partial}{\partial \xi}\left[\xi^{3/2}\, D_{ql}(\xi,t)\, \frac{\partial f}{\partial \xi}\right],$$

$$\partial f \,/\, \partial t = 0, \quad \xi \to \infty \quad if \quad D_{ql}(\xi,t) = \tilde{D}(\xi)\cdot\tilde{D}(t)\,, \quad \tilde{D}(t) = T^{-1/2} \Rightarrow$$

$$\tilde{D}(f) = \frac{1}{\xi^{1/2}} \frac{\partial}{\partial \xi}\left[\xi^{3/2}\, \tilde{D}(\xi)\, \frac{\partial f}{\partial \xi}\right], \quad \tilde{D}(\xi) = D_0 \xi^{p}\,, \quad 1 \geq p \geq 0$$

$$p = 0 \to \quad f(\xi,t) \propto \exp\{-\xi\}, \qquad \textit{Maxwell distribution}$$

Strongly enhanced tails

$$F(\xi \to \infty) \propto \exp\left\{ -const \cdot \frac{\xi^{1-p}}{(1-p)} \right\}; \quad 1 > p > 0$$

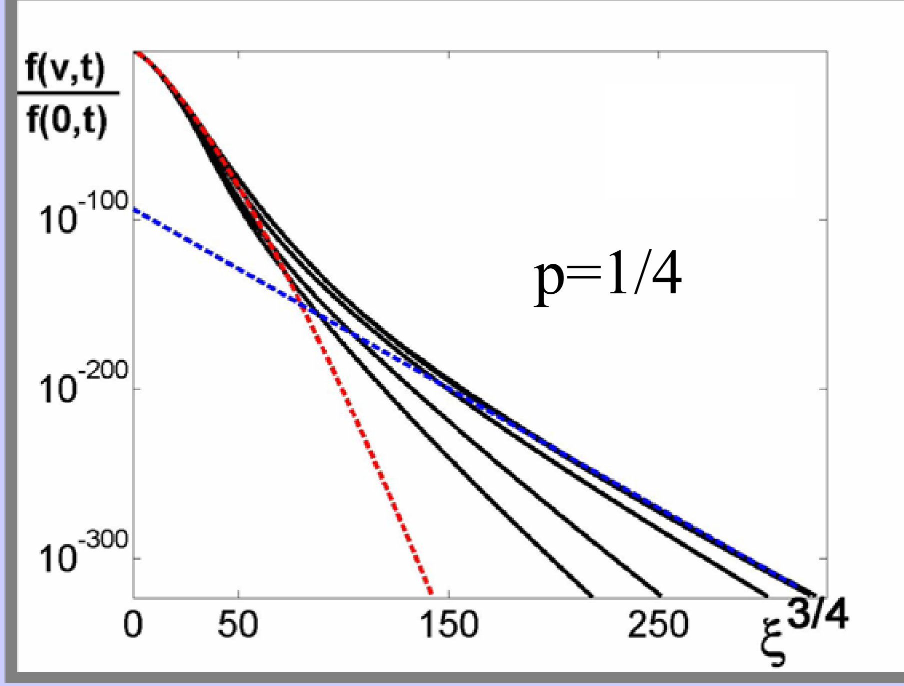
$$F(\xi \to \infty) \propto \xi^{-5/2},$$

$\boxed{p=1}$

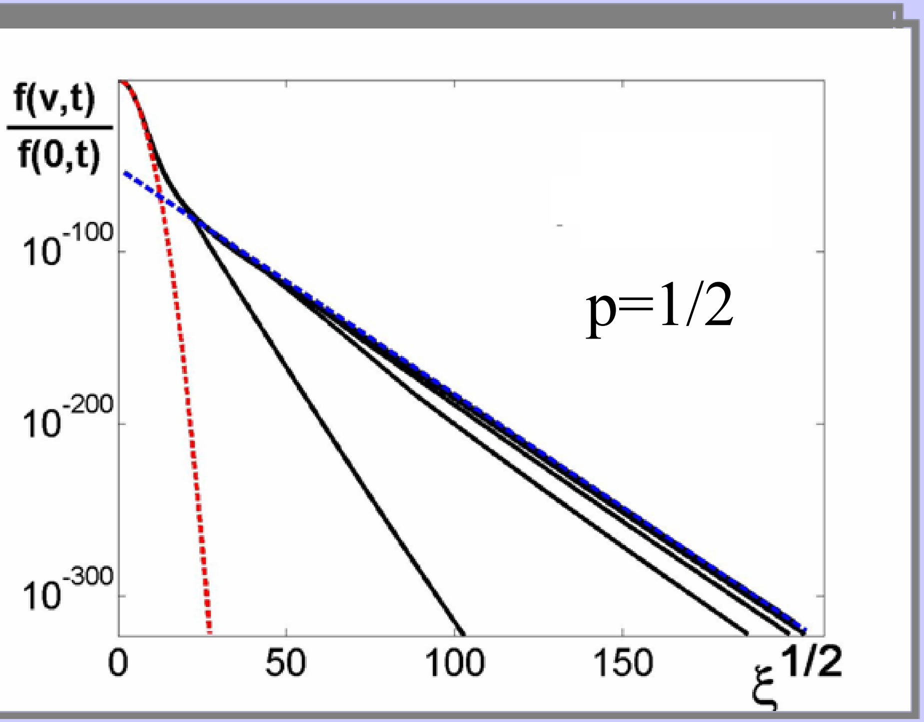
Numerically: the Cauchy problem

Finite –difference schemes

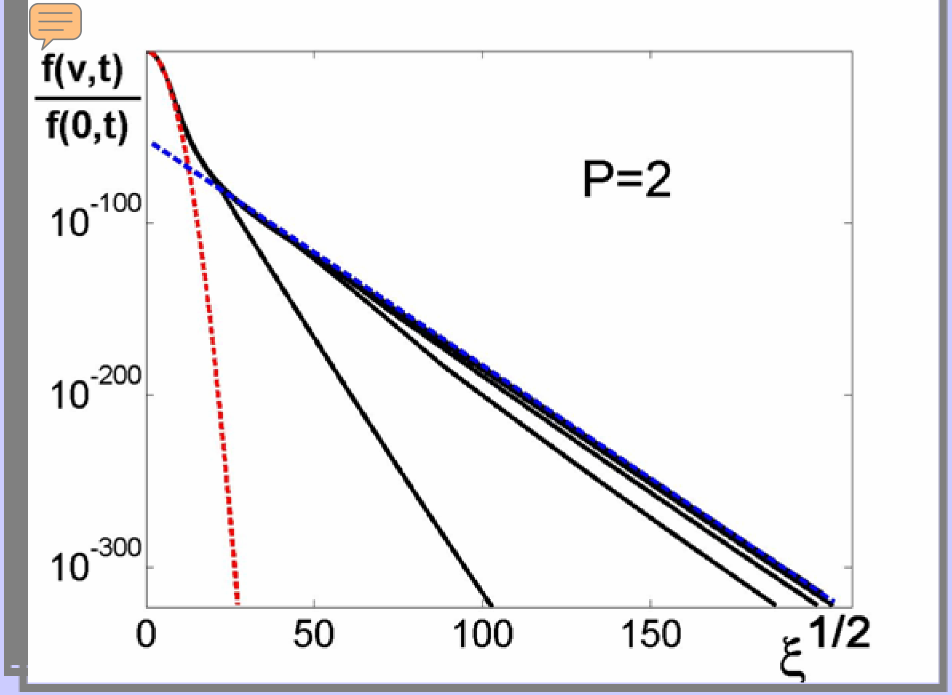
STOCHASTIC MODELING OF THE QUASILINEAR DIFFUSION



f(v,t)
f(0,t)
10^-100
10^-200
10^-300
p=1/4
0
50
150
250
ξ^3/4



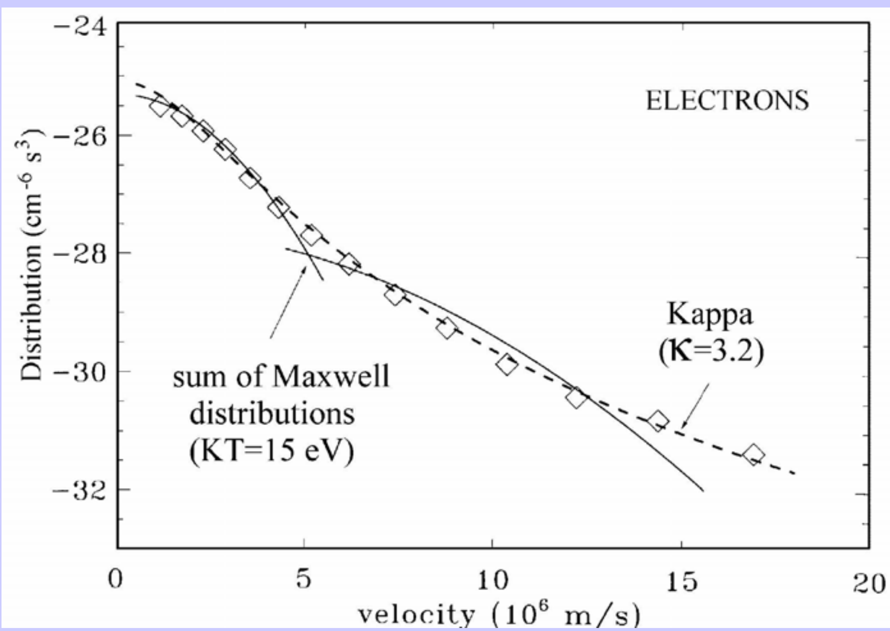
f(v,t)
f(0,t)
10^-100
10^-200
10^-300
p=1/2
0
50
100
150
ξ^1/2

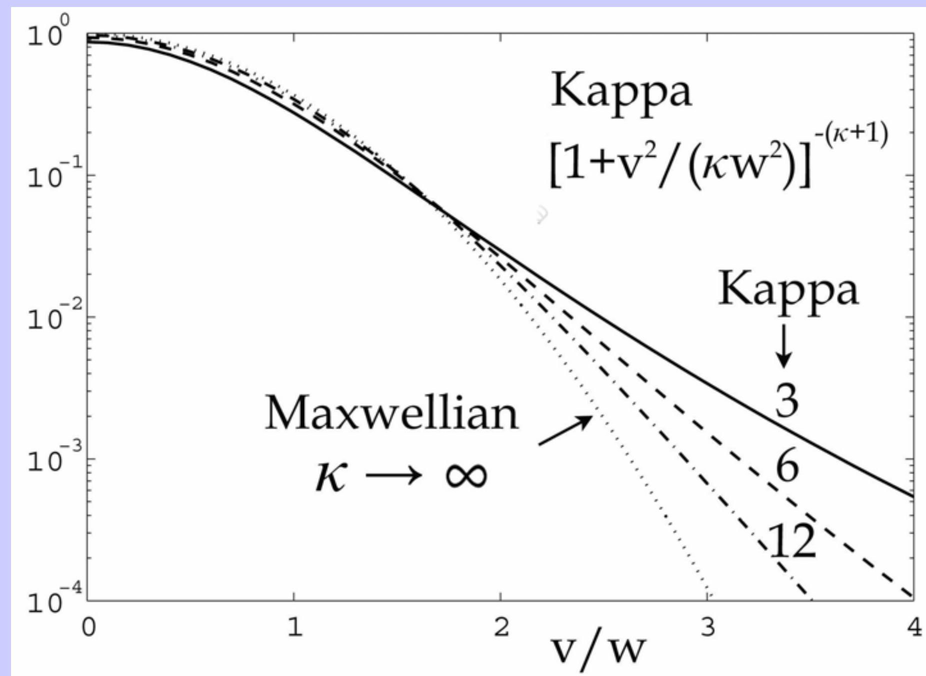


Nicole Meyer-Vernet *

Planetary and Space Science 49 (2001) 247–260

Solar wind electrons





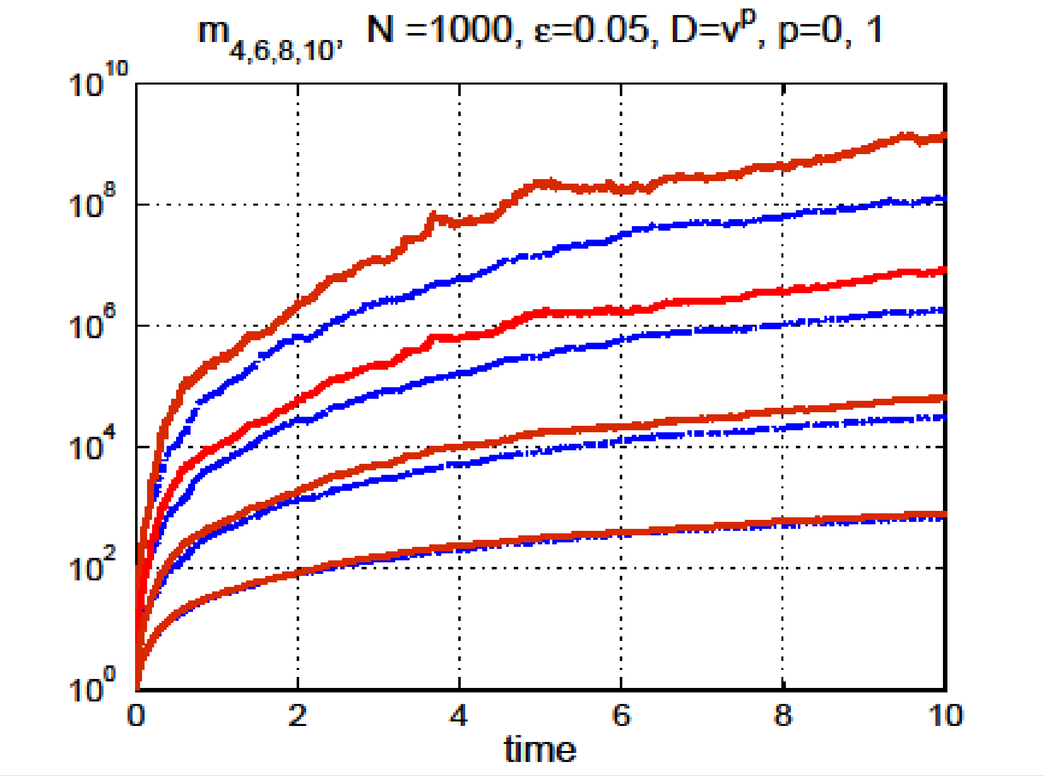
Резюме

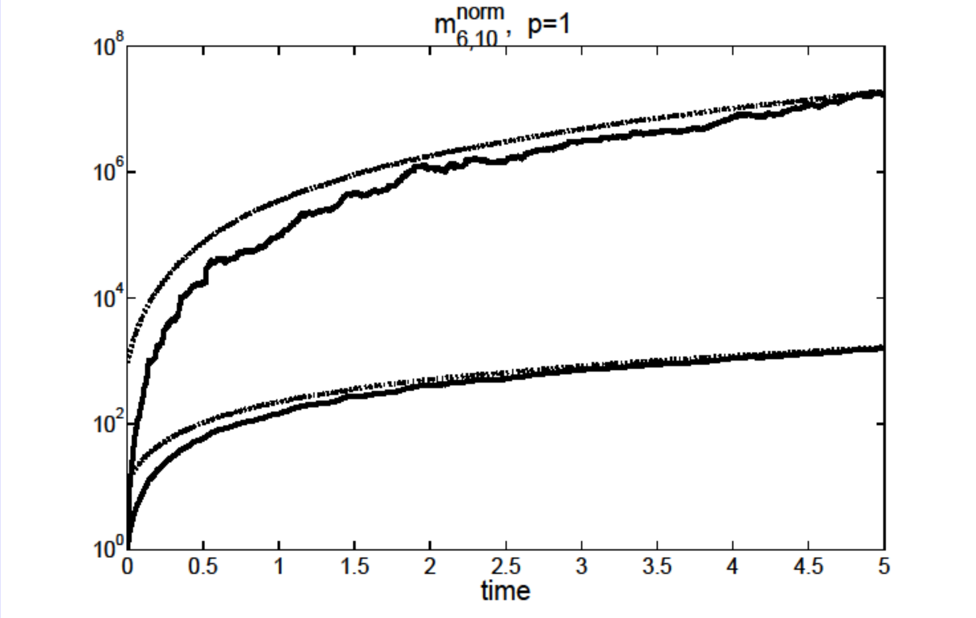
Предложен новый DSMC метод моделирования интеграла столкновений для потенциалов с бесконечным радиусом действия.

Для кинетического уравнения ЛФП с локализованным в пространстве скоростей нагревом H(f) решения имеют сильно обеднённый хвост в сравнении с максвелловским распределением.

Оператор диффузии с расширенным классом коэффициентов квазилинейной диффузии, приводит к существенному ускорению частиц (увеличению хвоста распределения).

Стохастическое моделирование задачи верифицируется детерминистическим подходом (асимптотическими аналитическими результатами) и наоборот.





Comparison of the numerical moments with the analytical moments

Tsallis Statistics

$$S_q(p) = \frac{1}{q-1}\left(1 - \int (p(x))^q \, dx\right),$$

Non-additivity

Given two independent systems *A* and *B*, for which the joint probability density satisfies

$$p(A,B) = p(A)p(B),$$

the Tsallis entropy of this system satisfies

$$S_q(A,B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B).$$

From this result, it is evident that the parameter $|1 - q|$ is a measure of the departure from additivity. In the limit when $q = 1$, $S(A,B) = S(A) + S(B),$ which is what is expected for an additive system.

Kappa distribution

$$\lim_{n\to\infty}\left[1+\frac{v^2}{\kappa v_e^2}\right]^{-\kappa-1} = \exp\left(-\frac{v^2}{v_e^2}\right)$$